Half-Life Bias Correction and the G7 Stock Markets

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Nov. 16, 2008

Abstract

We look at alternative methods to correct a downward bias in half-life estimates of relative stock prices among the G7 countries. We compare a grid-α median-unbiased method (GA) and a recursive mean adjustment (RMA) method. The confidence intervals for GA half-life estimates are all infinite while RMA confidence intervals are frequently compact. RMA unit root tests also reject the null for more countries than standard ADF or GA methods.

Keywords: International Equity Prices; Recursive Mean Adjustment; Unit Root Process

JEL Classification: C22, G15

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I. Introduction

Since the seminal work of Kendall (1954), the time series literature has long recognized the potential bias present in least squares (LS) estimation of the half-life for a mean-reverting process. The bias can become potentially severe as one nears a unit root. Consequently, several methods have been proposed to correct this bias, the most popular being the median-unbiased methods of Andrews (1993), Andrews and Chen (1994), and Hansen (1999). Median-unbiased methods and traditional augmented Dickey-Fuller (ADF) unit root tests have been commonly applied to international and domestic financial data with somewhat mixed results.[1]

An alternative approach, based on recursive mean adjustment, has also been put forth by Shin and So (1999, 2001). Despite the superior power[2] and computational simplicity of their approach, the financial literature has largely overlooked recursive mean adjustment (RMA) methods. We seek to fill this gap by re-examining the issue of mean-reversion between the major G7 stock markets using RMA methods. Our setup is identical to that in Balvers et al. (2000) and we also employ traditional ADF and median-unbiased methods for comparison. The empirical conclusions are strikingly different depending on the chosen methodology and the RMA approach displays superior power and precision.

The remainder of this letter is organized as follows. The second section specifies our econometric model and outlines the basic RMA and median-unbiased methodologies. The third section contains a description of the data and the empirical results. The last section concludes.

II. The Econometric Model

Let p_t^i and p_t^j be the natural logarithm of the stock price index for country i and j, respectively. When p_t^i and p_t^j share a unique unit root component, the deviation of p_t^i relative to p_t^j, denoted as r_t, should follow a mean-reverting process. That is,

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[1] For example, see Balvers et al. (2000), Chaudhuri and Wu (2004), and Gropp (2006).
[2] See Taylor (2002).

$$r_t = c + \alpha r_{t-1} + u_t, \tag{1}$$

where u_t is a mean-zero white noise process. It is well-known that the LS estimator $\hat{\alpha}_{LS}$ from (1) can be equivalently estimated by the following equation:

$$\left(r_t - \bar{r}\right) = \alpha\left(r_{t-1} - \bar{r}\right) + u_t, \tag{2}$$

where $\bar{r} = (1/n)\sum_{s=1}^{n} r_s$.

The $\hat{\alpha}_{LS}$ is biased since u_t is correlated with the regressor, $r_{t-1} - (1/n)\sum_{s=1}^{n} r_s$, for $t \le s$. Subsequent to the seminal work of Kendall (1954)[3], a variety of methods have been put forward to correct this bias, such as Andrews (1993), Andrews and Chen (1994), and Hansen (1999). In this paper, we correct for the bias by employing the recursive mean adjustment (RMA) estimator proposed by So and Shin (1999). This relatively new method has received very little attention in the econometrics literature, yet it is computationally simple and possesses greater power than alternative methods.[4]

For the RMA estimator, rewrite (1) as follows:

$$r_t - \bar{r}_{t-1} = \alpha(r_t - \bar{r}_{t-1}) + \varepsilon_t, \tag{3}$$

where $\bar{r}_{t-1} = (t-1)^{-1}\sum_{s=1}^{t-1} r_s$ and $\varepsilon_t = u_t - (1-\alpha)(t-1)^{-1}\sum_{s=1}^{t-1} r_s$. Note that the regressor ($r_t - \bar{r}_{t-1}$) is independent of the error term ε_t , which results in bias reduction for the RMA estimator,

$$\hat{\alpha}_{RMA} = \frac{\sum_{t=2}^{n}(r_{t-1} - \bar{r}_{t-1})(r_t - \bar{r}_{t-1})}{\sum_{t=2}^{n}(r_{t-1} - \bar{r}_{t-1})^2} \tag{4}$$

Extending the RMA estimation to higher order autoregressive processes is straightforward. When u_t is serially correlated, (1) can be rewritten as the following augmented Dickey-Fuller (ADF) regression equation:

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[3] His method can be used to obtain an exactly mean-unbiased estimate for α when the true data generating process is AR(1).

[4] Thanks to reduced-bias estimation, the left p[th] percentile shifts to the right, while asymptotic distributions of the RMA and LS estimators are identical. This leads to power improvement (see So and Shin, 1999) from the LS estimator.

$$r_t = c + \alpha r_{t-1} + \sum_{j=1}^{k} \beta_j \Delta r_{t-j} + \eta_t, \quad (5)$$

where η_t is a mean-zero white noise process. By regressing r_t on $(\Delta r_{t-1}, \Delta r_{t-2}, ..., \Delta r_{t-k})'$, we obtain the following:[5]

$$r_t^* = c + \alpha r_{t-1} + \eta_t^*, \quad (6)$$

where $r_t^* = r_t - \sum_{j=1}^{k} \hat{\beta}_{j,LS} \Delta r_{t-j}$ and $\eta_t^* = \eta_t + \sum_{j=1}^{k} (\beta_j - \hat{\beta}_{j,LS}) \Delta r_{t-j}$. Then, the RMA technique described by (3) and (4) applies to obtain $\hat{\alpha}_{RMA}$.

We also employ Hansen's (1999) method for (median) bias correction[6] for purposes of comparison. Hansen's method can be implemented by defining the following grid-α statistic at each of M grid points $\alpha_j \in [\alpha_1, \cdots, \alpha_M]$ around the neighborhood of $\hat{\alpha}_{LS}$ from the ADF regression equation (5):

$$b(\alpha_j) = \hat{\alpha}_{LS} - \alpha_j. \quad (7)$$

For each grid point α_j, we first run LS estimations for β s by regressing ($r_t - \alpha_j r_{t-1}$) on $(\Delta r_{t-1}, \Delta r_{t-2}, ..., \Delta r_{t-k})'$. Note that we assume that each grid point α_j as a true value for α and that the resulting $\hat{\beta}_{LS}$ are treated as nuisance parameters that are functions of α_j.

Next, we generate B set of pseudo observations for each α_j and implement LS estimations for these bootstrap samples to obtain the (p quantile) grid-α bootstrap quantile function estimates,

$$\hat{\psi}_{N,p}^*(\alpha_j) = \hat{\psi}_{N,p}^*(\alpha_j, \beta(\alpha_j)), \quad (8)$$

where N is the number of observations in each of B set of pseudo observations. We obtain the median unbiased estimate by,

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[5] Choi et al. (2008) also employ a similar method in a panel framework.
[6] Hansen's method is equivalent to Andrews (1993) and Andrews and Chen (1994) if the error term is normally distributed.

$$\hat{\alpha}_{MUE} = \alpha_j \text{, s.t. } b(\alpha_j) = \tilde{\psi}^*_{N,50\%}(\alpha_j), \quad (9)$$

where $\tilde{\psi}^*_{N,50\%}(\alpha_j)$ denotes the smoothed quantile function estimates.[7] For the 95% confidence band,

$$\hat{\alpha}_L = \alpha_j \text{, s.t. } b(\alpha_j) = \tilde{\psi}^*_{N,97.5\%}(\alpha_j) \text{ and } \hat{\alpha}_U = \alpha_j \text{, s.t. } b(\alpha_j) = \tilde{\psi}^*_{N,2.5\%}(\alpha_j), \quad (10)$$

where $\hat{\alpha}_L$ and $\hat{\alpha}_U$ are the 95% lower and upper end of the confidence interval, respectively.

III. Data and Results

We utilize MSCI gross end-of-period monthly data on stock indices from Canada, France, Germany, Italy, and the U.K. and compute their deviations from the U.S. stock index over the period of December, 1969 through September, 2007. The U.S. serves as the reference country in our analysis and all indices are, therefore, expressed in U.S. dollars. Our data and basic setup follows Balvers et al. (2000), although we utilize monthly data and focus on the G7 countries.

Three unit-root tests are performed on each country in our sample. The first two tests are the traditional augmented Dickey-Fuller (ADF) test and a generalized least squares ADF test (DFGLS).[8] The third test is an ADF-type statistic based on the RMA methods outlined in section II (ADF_{RMA}).[9] The results from all three unit root tests are presented in Table 1.

Table 1 indicates that the RMA-based unit root test yields the most rejections of the null, with only Canada and Japan failing to reject. The standard ADF test rejects the null for only one country, Italy. The (weak) rejection of the null by the DFGLS test for Canada is somewhat disturbing as Canadian data is usually troublesome (approximately a unit root) due to the heavy influence of commodity prices on their exchange rate.[10] This perhaps confirms the recent findings of instability in the DFGLS test due to initial condition sensitivity.[11] Table 2 presents estimates of half-life and 95%-confidence intervals

[7] We use the Epanechinikov kernel, following Hansen (1999).
[8] See Elliot et al. (1996).
[9] See So and Shin (2001).
[10] Since we use deviation of stock prices measured in the U.S. dollar, they are influenced by the exchange rates.
[11] See Choi and Moh (2006).

computed using standard least squares (LS), a grid-α median-unbiased (GA) method, and a recursive mean adjustment (RMA) method.

Table 1. Unit Root Test Results

Country	ADF	DFGLS	ADF_{RMA}
Canada	-1.612	-1.640*	-1.085
France	-2.506	-1.875*	-1.915**
Germany	-2.321	-1.891*	-1.638*
Italy	-2.877**	-0.887	-2.077**
Japan	-1.879	-0.654	-0.724
UK	-2.505	-1.464	-1.774*

Note: i) ADF refers the augmented Dickey-Fuller *t*-statistics when an intercept is included. ii) ADF_{RMA} refers the recursive mean adjustment unit root test statistics when an intercept is included. iii) The number of lags (*k*) is chosen by the general-to-specific rule (Hall, 1994) with the maximum 12 lags for the ADF and ADF_{RMA} tests. For the DFGLS test, the modified Akaike Information Criteria (MAIC) was used as recommended by Ng and Perron (2001). iii) * and ** refer the case when the unit root null is rejected at the 10% and 5% significance level, respectively.
iv) The asymptotic critical values for the DFGLS are from Ng and Perron (2001), and from Shin and So (2001) for the ADF_{RMA}.

Table 2. Half-Life Estimates and 95% Confidence Interval

Country	LS		Grid-α		RMA	
Canada	7.254	[1.574,15.02]	35.43	[3.974, ∞)	11.12	[1.862, ∞)
France	1.971	[0.784,3.330]	2.765	[1.257, ∞)	2.600	[0.878,5.104]
Germany	2.156	[0.743,3.821]	3.242	[1.309, ∞)	3.272	[0.829,6.583]
Italy	2.301	[0.892,3.986]	2.919	[1.450, ∞)	3.292	[1.004,6.187]
Japan	6.510	[1.551,13.71]	21.75	[3.606, ∞)	17.34	[1.847, ∞)
UK	2.096	[0.784,3.693]	2.902	[1.292, ∞)	2.976	[0.880,5.846]

Note: i) The 95% least squares confidence intervals were obtained by taking 2.5 and 97.5% percentiles from 10,000 nonparametric bootstrap simulations at the least squares point estimates (Efron and Tibshirani, 1993). ii) The 95% grid-α confidence intervals were obtained from 10,000 nonparametric bootstrap simulations at the 30 grid points in the neighborhood of the least squares point estimates (Hansen, 1999). iii) The 95% recursive mean adjustment confidence intervals were obtained by taking 2.5 and 97.5% percentiles from 10,000 nonparametric bootstrap simulations at the RMA point estimates (Efron and Tibshirani, 1993, So and Shin, 1999).

The half-life estimates in Table 2 display two interesting features. First, the LS estimates are significantly downward-biased compared to the GA and RMA estimates. More importantly, there is a remarkable difference in the compactness of the confidence intervals between the GA and RMA methods. While four of the countries have finite confidence intervals with the RMA method, all of the countries have infinite confidence intervals when the median-unbiased (GA) method is employed. The RMA method yields superior precision in terms of half-life estimation when compared to median-unbiased methods.

IV. Conclusions

We examined the mean reversion between the G7 stock markets using alternative methods of half-life bias correction. The often overlooked RMA methods of Shin and So (1999, 2001) exhibit greater power and precision in terms of rejecting unit roots and estimating half-life confidence intervals. Four of the G7 countries are found to exhibit mean reversion and possess compact half-life confidence intervals when RMA methods are employed. In contrast, a standard ADF test and median-unbiased (GA) methods reject a unit root for only one country and none of the countries are found to possess a compact confidence interval for the half-life. This rather striking differential in empirical results suggests that the use of recursive mean adjustment methods can yield substantially different conclusions in practical applications where the underlying data are near unit root processes.

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