

Half-Life Estimation under the Taylor Rule*

Hyeongwoo Kim^{†‡}

The Ohio State University

Job Market Paper

July 2005

Abstract

This paper addresses two perennial problems in the existing Purchasing Power Parity (PPP) literature, namely, unreasonably long half-life estimates of PPP deviations (Rogoff, 1996) and extremely wide confidence intervals for half-life point estimates (Murray and Papell, 2002). One answer to these problems can be found in the work of Kim, Ogaki, and Yang (2003), who used a system method and estimated much shorter half-lives than Rogoff's "Remarkable Consensus" of 3 to 5 year half life, though with limited efficiency improvements. Rather than following these authors in using a money demand function, we incorporated a forward-looking version of the Taylor Rule into a system of exchange rates and inflation. In so doing, we obtained a substantial improvement in efficiency, as well as reasonably short half-lives for PPP deviations. Our model also indicates that real exchange rate dynamics may differ greatly, depending on the pattern of systematic central bank responses to the inflation rate.

Keywords: Purchasing Power Parity, Taylor Rule, Half-Life of PPP Deviations

JEL Classification: C32, E52, F31

*Earlier version of this paper was presented in 14th Annual Midwest Econometrics Group Meeting (2004) with the title "Purchasing Power Parity under a Taylor Rule Type Monetary Policy."

[†]Correspondence: Hyeongwoo Kim, Department of Economics, the Ohio State University, Columbus, OH 43210. Tel: (614) 292-2070. Fax: (614) 292-3906. Email: kim.1423@osu.edu.

[‡]I am grateful to Jaebeom Kim, Paul Evans, J. Huston McCulloch, Pok-sang Lam, Bill Dupor, and seminar participants in 14th Midwest Econometrics Group meeting for helpful suggestions. Special thanks are due to Masao Ogaki for valuable comments and warm encouragement. The use of the GMM package by Hansen, Heaton, and Ogaki (1993) is gratefully acknowledged.

1 Introduction

Since its revival by Cassel (1921), Purchasing Power Parity (PPP) has been one of the most useful building blocks in monetary models (e.g., Frenkel, 1976) and neoclassical models of exchange rates (e.g., Lucas, 1982), as well as in the more recent Redux model (Obstfeld and Rogoff, 1995). This paper attempts to provide a solution to two major problems that have frequently arisen in recent empirical PPP literature. These problems may be summarized as follows.¹

Rogoff (1996) has described the “PPP puzzle” as the question of how one might reconcile highly volatile short-run movements of real exchange rates with an extremely slow convergence rate to PPP, and alluded to the “remarkable consensus” of three- to five-year half-life estimates for PPP deviations from long-horizon data². In other words, though financial factors, such as monetary shocks, may successfully account for the short-run volatility of real exchange rates in the presence of nominal rigidities, it is hard to rationalize why such PPP deviations should attenuate that slowly, if such shocks truly are largely neutral in the medium-run.³

Another issue, raised by Murray and Papell (2002), concerns the possibility of conducting a meaningful statistical inference of the length of half-lives of real exchange rates. Their half-life estimates for many real exchange rates turned out to have extremely wide confidence intervals, even though their point estimates were consistent with Rogoff’s remarkable consensus.

It should be noted that most of the researchers named above acquired their half-life estimates using a univariate process model of real exchange rates⁴. It would seem, therefore, that such a single-equation approach does not facilitate a resolution of these issues. Interestingly, recently formulated panel approaches have tended to provide relatively shorter half-lives for current float data (e.g., 2.5-

¹Due to the frequent failure of the law of one price in microeconomics data (e.g. Isard 1977), very few economists consider PPP as a short-run proposition. The empirical evidence on the validity of PPP in the long-run is mixed (see Rogoff 1996 for a survey). This paper accepts PPP as a long-run proposition without further discussing this issue.

²More recently, Taylor (2002) report shorter half-life estimates than this consensus half-life from over 200-year long data for 20 countries. However, Lopez, Murray, and Papell (2004) claim his results are not robust to lag selection problems.

³Real shocks such as technology shocks may account for such a slow convergence rate, since those shocks may not be neutral even in the long-run. However, real shocks may not be able to successfully explain the short-run volatility of real exchange rates.

⁴If PPP holds in the long-run, the real exchange rate s_t ($e_t + p_t^* - p_t$) may be represented as the following simple AR(1) process, $s_t = \mu + \alpha s_{t-1} + \varepsilon_t$, where $\alpha < 1$. Then, the corresponding half-life can be calculated as $\ln(.5)/\ln \alpha$.

year half-lives, Papell, 1997, and Wu, 1996). Moreover, Murray and Papell (2005) obtained reasonably compact confidence intervals for their half-life panel estimates. Their estimates, however, confirmed Rogoff’s consensus half-life. Therefore, panel approaches have been insufficient as a solution to the PPP puzzle, even though they have provided some efficiency gains. Furthermore, the main assumption of the panel approach, that the convergence rates to PPP were the same across countries, is hard to justify. Recently, Imbs et. al. (2005) showed that ignoring heterogeneous dynamics may result in quite inaccurate half-life estimates⁵.

One potential answer to these problems can be found in the work of Kim, Ogaki, and Yang (2003). Instead of single-equation approaches, they suggested using a system method that combines economic theories with the single-equation model of real exchange rates. Imposing theory-driven restrictions on their model of exchange rates and inflation, they estimated half-lives that were much shorter⁶ than the current consensus of three- to five-year half-lives for the post Bretton Woods CPI-based real exchange rates. It should be noted that one advantage of using such a system method is that it provides estimates that are more efficient than those from a single-equation method, as long as the imposed restrictions are valid. Indeed, these authors obtained limited efficiency gains, which may have allowed more suitable statistical inferences to be made from their point estimates.

The research presented in this paper constitutes a modification of the work of Kim, Ogaki, and Yang (2003), who derived restrictions from the conventional money-market equilibrium condition, which they identified using money demand functions. However, the money demand function can be quite unstable, especially in the short-run, so their use of the money-market equilibrium condition may not have been ideal. In contrast, we incorporated a forward-looking version of the Taylor Rule into a dynamic system of the exchange rate and inflation. In so doing, we attempted to determine whether Taylor Rule-based restrictions could resolve the problems noted above. In fact, we obtained a substantial efficiency improvement over the results of Kim, Ogaki, and Yang (2003), and our point estimates were reasonably shorter (1.49- and 1.37-year median half-lives for GDP deflator- and CPI-

⁵Chen and Engel (2005) claim that such aggregation bias may not have significant impacts on panel approaches.

⁶Their half-life estimates range from 0.12 to 2.22 years. They don’t report the median half-life, but it was 0.35 year from full sample estimations by our calculation.

based real exchange rates, respectively) than the current consensus of three- to five-year half-lives. Interestingly, our estimates are roughly consistent with those of Crucini and Shintani (2004), who reported 1.1-, 1.0-, and 1.6-year baseline half-life estimates for OECD micro data on all, traded, and non-traded good prices, respectively⁷.

Since the seminal work of Taylor (1993), the Taylor Rule has been one of the most popular models used in the monetary policy literature. The core implication of the Taylor Rule is that the price level would be indeterminate unless the central bank responds to inflation aggressively enough to raise the real interest rate.

One particularly interesting point has been made by Clarida, Galí and Gertler (2000), who provided strong empirical evidence of a structural break in the Fed's reaction function⁸. Putting it differently, they found that the estimate of the coefficient on rationally expected near-future inflation became strictly greater than one for the period of the Volker-Greenspan era, whereas the corresponding estimate for the pre-Volker era turned out to be strictly less than one; these results are consistent with the implications of the Taylor Rule and observed inflation dynamics⁹. Similar findings have been provided by Taylor (1999a) and Judd and Rudebusch (1998). Clarida, Galí and Gertler (1998) also found similar international evidence for Germany and Japan.

This paper shows that consideration of such a structural break may be important in understanding real exchange rate dynamics. It turns out that the dynamics of real exchange rates can differ greatly, depending on the pattern of systematic central bank responses to inflation. In what follows, we show that exchange rate dynamics can be greatly affected by the present value of rationally expected *future* fundamental variables only when the inflation coefficient is strictly greater than one. Interestingly, when the inflation coefficient is less than one, real exchange rate dynamics can be explained only by *past* economic variables and *any* martingale difference sequences; moreover, future fundamental variables play no role.

⁷Parsley and Wei (2004) also provide similar micro-evidence.

⁸Unlike Taylor (1993), they used a forward looking version of the Taylor rule. In their model, the Fed is assumed to respond to future inflation forecast rather than current inflation.

⁹One can observe rapidly rising inflation rates in the pre-Volker era and declining inflation rates in the Volker-Greenspan era.

This paper is not the first to emphasize the importance of expected future (Taylor Rule) fundamentals in the context of real exchange rate dynamics. Engel and West (2002) drew similar conclusions using data for the US and Germany. However, they did not attempt to estimate the half-lives of PPP deviations. In addition, Mark (2005) presented similar evidence that the dynamics of the real US\$-Deutschemark exchange rate could be better understood by means of Taylor Rule fundamentals in a learning framework¹⁰.

The rest of the paper is organized as follows. In Section 2, we construct a system of stochastic difference equations for the exchange rate and inflation, explicitly incorporating a forward-looking version of the Taylor Rule into the system. Then, we describe three different estimation strategies. These include a GMM system method, which combines the single-equation method with the model-driven restriction from the Taylor Rule. In Section 3, a description of the data and the estimation results are provided. Section 4 concludes.

2 Model Specification

2.1 Gradual Adjustment Equation

We start with a simple univariate stochastic process of real exchange rates. Let p_t be the log domestic price level, p_t^* be the log foreign price level, and e_t be the log nominal exchange rate as the price of one unit of the foreign currency in terms of the home currency. And we denote s_t as the log of the real exchange rate, $p_t^* + e_t - p_t$.

Rather than econometrically testing it¹¹, we simply assume that PPP holds in the long-run¹². Putting it differently, we assume that there exists a cointegrating vector $[1 \ -1 \ -1]'$ for a vector $[p_t \ p_t^* \ e_t]'$, where p_t , p_t^* , and e_t are difference stationary processes. Under this assumption, real

¹⁰Obstfeld (2004) discusses implications of optimal interest rate rules on exchange rate regimes in a dynamic stochastic general equilibrium model. For a similar discussion, see Devereux and Engel (2005).

¹¹The empirical evidence on the validity of PPP in the long-run is mixed. It should be noted that even when we have some evidence against PPP, such results might be due to lack of power of existing unit root tests in small samples, and are subject to the observational equivalence problem.

¹²We may assume that PPP applies only to the tradable price-based real exchange rates by distinguishing tradables from non-tradables. See Kim and Ogaki (2004) or Kim (2005a) for details.

exchange rates can be represented as the following stationary univariate autoregressive process of degree one.

$$s_{t+1} = d + \alpha s_t + \varepsilon_{t+1}, \quad (1)$$

where α is a positive persistence parameter that is less than one¹³.

Note that α can be consistently estimated by the conventional least squares method under the maintained cointegrating relation assumption¹⁴ as long as there's no measurement error¹⁵. Once we obtain the point estimate of α , the half-life of the real exchange rate can be obtained by $\frac{\ln(.5)}{\ln \alpha}$ ¹⁶.

Interestingly, Kim, Ogaki, and Yang (2003) pointed out that the equation (1) could be implied by the following error correction model of real exchange rates by Mussa (1982) with a *known* cointegrating relation described earlier.

$$\Delta p_{t+1} = b[\mu - (p_t - p_t^* - e_t)] + E_t \Delta p_{t+1}^* + E_t \Delta e_{t+1}, \quad (2)$$

where $\mu = E(p_t - p_t^* - e_t)$, $b = 1 - \alpha$, $d = -(1 - \alpha)\mu$, $\varepsilon_{t+1} = \varepsilon_{1t+1} + \varepsilon_{2t+1} = (E_t \Delta e_{t+1} - \Delta e_{t+1}) + (E_t \Delta p_{t+1}^* - \Delta p_{t+1}^*)$, and $E_t \varepsilon_{t+1} = 0$. $E(\cdot)$ denotes the unconditional expectation operator, and $E_t(\cdot)$ is the conditional expectation operator on I_t , the economic agent's information set at time t .

One interpretation of this equation is that the domestic price level adjusts instantaneously to the expected *change* in its PPP, while it adjusts to its unconditional PPP level, $E(p_t^* + e_t)$ only slowly with the constant convergence rate $b (= 1 - \alpha)$, which is a positive constant less than unity by construction¹⁷.

¹³Note that this is a so-called Dickey-Fuller estimation model. One may estimate half-lives by an Augmented Dickey-Fuller estimation model in order to avoid possible serial correlation problems. However, as shown in Murray and Papell (2002), half-life estimates from both models were roughly similar. So it seems that AR(1) specification is not a bad approximation.

¹⁴It is well-known that the standard errors obtained from the least squares or parametric bootstrap methods may not be valid asymptotically in presence of serially correlated errors. It would be necessary to use either Newey-West or QS kernel covariance estimators, or to use the moving block bootstrap method.

¹⁵If there is a measurement error problem, α may not be even consistent in this framework, since the aforementioned cointegrating relation assumption may not hold. We can deal with this problem by a two stage cointegration method that directly estimates the cointegrating vector rather than assuming it. See Kim, Ogaki, and Yang (2003) for details.

¹⁶Standard errors can also be obtained using the delta method.

¹⁷Recall that b (convergence rate parameter) equals to $1 - \alpha$, where α (persistence parameter) is a positive constant less than unity.

2.2 Taylor Rule Model

We assume that the uncovered interest parity holds. That is,

$$E_t \Delta e_{t+1} = i_t - i_t^*, \quad (3)$$

where i_t and i_t^* are domestic and foreign interest rates, respectively.

The central bank in the home country is assumed to continuously set its optimal target interest rate (i_t^T) by the following forward looking Taylor Rule¹⁸.

$$i_t^T = \iota + \gamma_\pi E_t \Delta p_{t+1} + \gamma_x x_t,$$

where ι is a constant that includes a certain long-run equilibrium real interest rate along with a target inflation rate¹⁹, and γ_π and γ_x are the *long-run* Taylor Rule coefficients on expected future inflation ($E_t \Delta p_{t+1}$) and current output deviations (x_t)²⁰, respectively. We also assume that the central bank attempts to smooth the interest rate by the following rule.

$$i_t = (1 - \rho) i_t^T + \rho i_{t-1},$$

that is, the current actual interest rate is a weighted average of the target interest rate and the previous period's interest rate, where ρ is the smoothing parameter. Then, we can derive the forward looking version Taylor Rule equation with interest rate smoothing policy as follows²¹.

$$i_t = \iota + (1 - \rho) \gamma_\pi E_t \Delta p_{t+1} + (1 - \rho) \gamma_x x_t + \rho i_{t-1} \quad (4)$$

¹⁸We do not impose any restriction on foreign central bank's behavior, since foreign variables are pure forcing variables in our model as will be illustrated later.

¹⁹See Clarida, Galí, and Gertler (1998, 2000) for details.

²⁰If we assume that the central bank responds to expected future output deviations rather than current deviations, we can simply modify the model by replacing x_t with $E_t x_{t+1}$. However, this does not make any significant difference to our results.

²¹It is also straightforward to modify our model where the home central bank employs the exchange rate targeting. See Kim (2005b) for reference.

Combining (3) and (4), we obtain the following.

$$\begin{aligned} E_t \Delta e_{t+1} &= \iota + (1 - \rho) \gamma_\pi E_t \Delta p_{t+1} + (1 - \rho) \gamma_x x_t + \rho i_{t-1} - i_t^* \\ &= \iota + \gamma_\pi^s E_t \Delta p_{t+1} + \gamma_x^s x_t + \rho i_{t-1} - i_t^*, \end{aligned} \quad (5)$$

where $\gamma_\pi^s = (1 - \rho) \gamma_\pi$ and $\gamma_x^s = (1 - \rho) \gamma_x$ are *short-run* Taylor Rule coefficients.

Now, let's rewrite (2) as the following equation in level variables.

$$p_{t+1} = b\mu + E_t e_{t+1} + (1 - b)p_t - (1 - b)e_t + E_t p_{t+1}^* - (1 - b)p_t^* \quad (2')$$

Taking differences and rearranging it, (2') can be rewritten as follows.

$$\Delta p_{t+1} = E_t \Delta e_{t+1} + \alpha \Delta p_t - \alpha \Delta e_t + [E_t \Delta p_{t+1}^* - \alpha \Delta p_t^* + \eta_t], \quad (6)$$

where $\alpha = 1 - b$ and $\eta_t = \eta_{1,t} + \eta_{2,t} = (e_t - E_{t-1}e_t) + (p_t^* - E_{t-1}p_t^*)$.

From (4), (5), and (6), we construct the following system of stochastic difference equations.

$$\begin{bmatrix} 1 & -1 & 0 \\ -\gamma_\pi^s & 1 & 0 \\ -\gamma_\pi^s & 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta p_{t+1} \\ E_t \Delta e_{t+1} \\ i_t \end{bmatrix} = \begin{bmatrix} \alpha & -\alpha & 0 \\ 0 & 0 & \rho \\ 0 & 0 & \rho \end{bmatrix} \begin{bmatrix} \Delta p_t \\ \Delta e_t \\ i_{t-1} \end{bmatrix} + \begin{bmatrix} E_t \Delta p_{t+1}^* - \alpha \Delta p_t^* + \eta_t \\ \iota + \gamma_x^s x_t - i_t^* \\ \iota + \gamma_x^s x_t \end{bmatrix} \quad (7)$$

For notational simplicity, let's rewrite (7) in matrix form as follows.

$$\mathbf{A} E_t \mathbf{y}_{t+1} = \mathbf{B} \mathbf{y}_t + \mathbf{x}_t, \quad (7')$$

and thus²²,

$$\begin{aligned} E_t \mathbf{y}_{t+1} &= \mathbf{A}^{-1} \mathbf{B} \mathbf{y}_t + \mathbf{A}^{-1} \mathbf{x}_t \\ &= \mathbf{D} \mathbf{y}_t + \mathbf{c}_t, \end{aligned} \tag{8}$$

where $\mathbf{D} = \mathbf{A}^{-1} \mathbf{B}$ and $\mathbf{c}_t = \mathbf{A}^{-1} \mathbf{x}_t$. By eigenvalue decomposition, (8) can be rewritten as follows.

$$E_t \mathbf{y}_{t+1} = \mathbf{V} \mathbf{\Lambda} \mathbf{V}^{-1} \mathbf{y}_t + \mathbf{c}_t, \tag{9}$$

where $\mathbf{D} = \mathbf{V} \mathbf{\Lambda} \mathbf{V}^{-1}$ and

$$\mathbf{V} = \begin{bmatrix} 1 & 1 & 1 \\ \frac{\alpha \gamma_\pi^s}{\alpha - \rho} & 1 & 1 \\ \frac{\alpha \gamma_\pi^s}{\alpha - \rho} & 1 & 0 \end{bmatrix}, \quad \mathbf{\Lambda} = \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \frac{\rho}{1 - \gamma_\pi^s} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Premultiplying (9) by \mathbf{V}^{-1} and redefining variables,

$$\mathbf{E}_t \mathbf{z}_{t+1} = \mathbf{\Lambda} \mathbf{z}_t + \mathbf{h}_t, \tag{10}$$

where $\mathbf{z}_t = \mathbf{V}^{-1} \mathbf{y}_t$ and $\mathbf{h}_t = \mathbf{V}^{-1} \mathbf{c}_t$.

Note that, among non-zero eigenvalues in $\mathbf{\Lambda}$, α is between 0 and 1 by definition, while $\frac{\rho}{1 - \gamma_\pi^s}$ ($= \frac{\rho}{1 - (1 - \rho) \gamma_\pi}$) is greater than unity as long as $1 < \gamma_\pi < \frac{1}{1 - \rho}$. Therefore, if the long-run inflation coefficient γ_π is strictly greater than one²³, the system of stochastic difference equations (7) has a saddle path equilibrium, where rationally expected future fundamental variables enter in the exchange rate and inflation dynamics. On the contrary, if γ_π is strictly less than unity, which might be true in the pre-Volker era in the US, the system would have a purely backward looking solution, where the solution would be determined by *past* fundamental variables and *any* martingale difference sequences.

Assuming γ_π is strictly greater than one, we can show that the solution to (7) satisfies the following

²²It is straightforward to show that \mathbf{A} is nonsingular, and thus has a well-defined inverse.

²³The condition $\gamma_\pi < \frac{1}{1 - \rho}$ is easily met for all sample periods we consider in this paper.

relation (see Appendix for the derivation).

$$\begin{aligned} \Delta e_{t+1} = & \hat{i} + \frac{\alpha\gamma_\pi^s}{\alpha - \rho} \Delta p_{t+1} - \frac{\alpha\gamma_\pi^s}{\alpha - \rho} \Delta p_{t+1}^* + \frac{\alpha\gamma_\pi^s - (\alpha - \rho)}{\alpha - \rho} i_t^* \\ & + \frac{\gamma_\pi^s(\alpha\gamma_\pi^s - (\alpha - \rho))}{(\alpha - \rho)\rho} \sum_{j=0}^{\infty} \left(\frac{1 - \gamma_\pi^s}{\rho} \right)^j E_t f_{t+j+1} + \omega_{t+1}, \end{aligned} \quad (11)$$

where,

$$\hat{i} = \frac{\alpha\gamma_\pi^s - (\alpha - \rho)}{(\alpha - \rho)(\gamma_\pi^s - (1 - \rho))} i_t,$$

$$\begin{aligned} E_t f_{t+j} = & - [E_t i_{t+j}^* - E_t \Delta p_{t+1+j}^*] + \frac{\gamma_x^s}{\gamma_\pi^s} E_t x_{t+j} \\ = & - E_t r_{t+j}^* + \frac{\gamma_x}{\gamma_\pi} E_t x_{t+j}, \end{aligned}$$

$$\begin{aligned} \omega_{t+1} = & \frac{\gamma_\pi^s(\alpha\gamma_\pi^s - (\alpha - \rho))}{(\alpha - \rho)\rho} \sum_{j=0}^{\infty} \left(\frac{1 - \gamma_\pi^s}{\rho} \right)^j (E_{t+1} f_{t+j+1} - E_t f_{t+j+1}) \\ & + \frac{\gamma_\pi^s}{\alpha - \rho} \eta_{t+1} - \frac{\alpha\gamma_\pi^s - (\alpha - \rho)}{\alpha - \rho} v_{t+1}, \end{aligned}$$

and,

$$E_t \omega_{t+1} = 0$$

Or, (11) can be rewritten with full parameter specification as follows.

$$\begin{aligned} \Delta e_{t+1} = & \hat{i} + \frac{\alpha\gamma_\pi(1 - \rho)}{\alpha - \rho} \Delta p_{t+1} - \frac{\alpha\gamma_\pi(1 - \rho)}{\alpha - \rho} \Delta p_{t+1}^* + \frac{\alpha\gamma_\pi(1 - \rho) - (\alpha - \rho)}{\alpha - \rho} i_t^* \\ & + \frac{\gamma_\pi(1 - \rho)(\alpha\gamma_\pi(1 - \rho) - (\alpha - \rho))}{(\alpha - \rho)\rho} \sum_{j=0}^{\infty} \left(\frac{1 - \gamma_\pi(1 - \rho)}{\rho} \right)^j E_t f_{t+j+1} + \omega_{t+1} \end{aligned} \quad (11')$$

Here, f_t is a proxy variable that summarizes the fundamental variables such as foreign real interest rates (r_t^*) and domestic output deviations.

Note that if γ_π is strictly less than unity, the restriction in (11) may not be valid, since the system

would have a backward looking equilibrium rather than a saddle path equilibrium²⁴. Put it differently, exchange rate dynamics critically depends on the size of γ_π . As mentioned in the introduction, however, we have some supporting empirical evidence for such a requirement for the existence of a saddle path equilibrium, at least for the sample period we consider²⁵. So we believe that our specification would remain valid for our purpose in this paper.

2.3 GMM Estimation

We discuss three estimation strategies here: single equation estimation; GMM estimation by Hansen and Sargent (1980, 1982); GMM system estimation by Kim, Ogaki, and Yang (2003).

2.3.1 Single Equation Estimation

We start with the conventional single equation approach. For this, let's rewrite (2) as follows.

$$\Delta p_{t+1} = b [\mu - (p_t - p_t^* - e_t)] + \Delta p_{t+1}^* + \Delta e_{t+1} + \varepsilon_{t+1}, \quad (2'')$$

where $\varepsilon_{t+1} = \varepsilon_{1t+1} + \varepsilon_{2t+1} = (E_t \Delta e_{t+1} - \Delta e_{t+1}) + (E_t \Delta p_{t+1}^* - \Delta p_{t+1}^*)$ and $E_t \varepsilon_{t+1} = 0$. It is easy to see that (2'') implies (1). Then one can estimate convergence parameter b (or persistence parameter α) by the conventional least square method²⁶.

2.3.2 GMM Estimation by Hansen and Sargent (1980, 1982)

Our second estimation strategy deals with the equation (11) or (11'). The estimation of the equation (11) is a challenging task, since it has an infinite sum of rationally expected discounted future fundamental variables. Following Hansen and Sargent (1980, 1982), we will linearly project $E_t(\cdot)$ onto Ω_t ,

²⁴If the system has a purely backward looking solution, the conventional Vector Autoregressive (VAR) estimation method may apply.

²⁵In contrast to Clarida, Gali, and Gertler (1998, 2000), Taylor (1999a), and Judd and Rudebusch (1998), Orphanides (2001) found the estimates of the inflation coefficient γ_π to be consistently greater than one in both the pre- and post-Volker regimes. In any case, our model specification is still valid.

²⁶However, in empirical perspectives, it is highly likely that the error terms are serially correlated due to aggregation bias especially when we use low frequency data. This problem can be fixed by correcting standard errors using either the Newey-West covariance estimator or QS kernel estimator.

the econometrician's information set at time t , which is a subset of I_t . Denoting $\hat{E}_t(\cdot)$ as such a linear projection operator onto Ω_t , we can rewrite (11) as follows.

$$\begin{aligned} \Delta e_{t+1} = & \hat{i} + \frac{\alpha\gamma_\pi^s}{\alpha - \rho}\Delta p_{t+1} - \frac{\alpha\gamma_\pi^s}{\alpha - \rho}\Delta p_{t+1}^* + \frac{\alpha\gamma_\pi^s - (\alpha - \rho)}{\alpha - \rho}i_t^* \\ & + \frac{\gamma_\pi^s(\alpha\gamma_\pi^s - (\alpha - \rho))}{(\alpha - \rho)\rho} \sum_{j=0}^{\infty} \left(\frac{1 - \gamma_\pi^s}{\rho}\right)^j \hat{E}_t f_{t+j+1} + \xi_{t+1}, \end{aligned} \quad (12)$$

where

$$\xi_{t+1} = \omega_{t+1} + \frac{\gamma_\pi^s(\alpha\gamma_\pi^s - (\alpha - \rho))}{(\alpha - \rho)\rho} \sum_{j=0}^{\infty} \left(\frac{1 - \gamma_\pi^s}{\rho}\right)^j (E_t f_{t+j+1} - \hat{E}_t f_{t+j+1}),$$

and

$$\hat{E}_t \xi_{t+1} = 0,$$

by the law of iterated projections.

Rather than choosing appropriate instrumental variables that are in Ω_t ²⁷, we simply assume $\Omega_t = \{f_t, f_{t-1}, f_{t-2}, \dots\}$. This assumption would be an innocent one under the stationarity assumption of the fundamental variable, f_t , and it can greatly lessen the burden in our GMM estimation by significantly reducing the number of coefficients to be estimated.

Let's assume, for now, that f_t be a zero mean covariance stationary, linearly indeterministic stochastic process so that it has the following Wold representation.

$$f_t = c(L)\nu_t, \quad (13)$$

where $\nu_t = f_t - \hat{E}_{t-1}f_t$ and $c(L)$ is square summable. Assuming that $c(L) = 1 + c_1L + c_2L^2 + \dots$ is invertible, (13) can be rewritten as the following autoregressive representation.

$$b(L)f_t = \nu_t, \quad (14)$$

where $b(L) = c^{-1}(L) = 1 - b_1L - b_2L^2 - \dots$. Linearly projecting $\sum_{j=0}^{\infty} \left(\frac{1 - \gamma_\pi^s}{\rho}\right)^j E_t f_{t+j+1}$ onto Ω_t ,

²⁷Kim, Ogaki, and Yang (2003) use the foreign inflation rate (Δp_t^*) as a scalar instrument variable.

Hansen and Sargent (1980) show that the following relation holds.

$$\sum_{j=0}^{\infty} \delta^j \hat{E}_t f_{t+j+1} = \psi(L) f_t = \left[\frac{1 - (\delta^{-1} b(\delta))^{-1} b(L) L^{-1}}{1 - (\delta^{-1} L)^{-1}} \right] f_t, \quad (15)$$

where $\delta = \frac{1 - \gamma_\pi^s}{\rho}$.

For actual estimation, we assume that f_t can be represented by a finite order AR(r) process²⁸, that is, $b(L) = 1 - \sum_{j=1}^r b_j L^j$, where $r < \infty$. Then, it can be shown that the coefficients of $\psi(L)$ can be computed recursively (see Sargent 1987) as follows.

$$\psi_0 = (1 - \delta b_1 - \dots - \delta^r b_r)^{-1}$$

$$\psi_r = 0$$

$$\psi_{j-1} = \delta \psi_j + \delta \psi_0 b_j,$$

where $j = 1, 2, \dots, r$. Then, the GMM estimation of (11) can be implemented by simultaneously estimating following two equations.

$$\begin{aligned} \Delta e_{t+1} = & \hat{i} + \frac{\alpha \gamma_\pi^s}{\alpha - \rho} \Delta p_{t+1} - \frac{\alpha \gamma_\pi^s}{\alpha - \rho} \Delta p_{t+1}^* + \frac{\alpha \gamma_\pi^s - (\alpha - \rho)}{\alpha - \rho} i_t^* \\ & + \frac{\gamma_\pi^s (\alpha \gamma_\pi^s - (\alpha - \rho))}{(\alpha - \rho) \rho} (\psi_0 f_t + \psi_1 f_{t-1} + \dots + \psi_{r-1} f_{t-r+1}) + \xi_{t+1}, \end{aligned} \quad (16)$$

$$f_{t+1} = k + b_1 f_t + b_2 f_{t-1} + \dots + b_r f_{t-r+1} + \nu_{t+1}, \quad (17)$$

where k is a constant scalar^{29,30}, and $\hat{E}_t \nu_{t+1} = 0$.

²⁸We can use conventional Akaike Information criteria or Bayesian Information criteria in order to choose the degree of such autoregressive processes.

²⁹Recall that Hansen and Sargent (1980) assume a zero-mean covariance stationary process. If the variable of interest has a non-zero unconditional mean, we can either demean it prior to the estimation or include a constant but leave its coefficient unconstrained. West (1989) showed that the further efficiency gain can be obtained by imposing additional restrictions on the deterministic term. However, the imposition of such an additional restriction is quite burdensome, so we simply add a constant here.

³⁰In actual estimations, we normalized (16) by multiplying $(\alpha - \rho)$ to each side in order to reduce nonlinearity.

2.3.3 GMM System Estimation

The system method (GMM) estimation combines aforementioned two estimation strategies. That is, we implement a GMM estimation for the three equations (2''), (16), and (17) simultaneously. Rather than leaving α in (16) unrestricted as in the Hansen-Sargent method, we restrict it to satisfy (2'') or (1) at the same time. In so doing, we may be able to acquire further efficiency gains by using more information.

A GMM estimation, then, can be implemented by the following $2(p+2)$ orthogonality conditions.

$$E\mathbf{x}_{1,t}(s_{t+1} - d - \alpha s_t) = 0 \quad (18)$$

$$\hat{E}\mathbf{x}_{2,t-\tau} \begin{pmatrix} \Delta e_{t+1} - \hat{i} - \frac{\alpha\gamma_\pi^s}{\alpha-\rho} \Delta p_{t+1} + \frac{\alpha\gamma_\pi^s}{\alpha-\rho} \Delta p_{t+1}^* - \frac{\alpha\gamma_\pi^s - (\alpha-\rho)}{\alpha-\rho} i_t^* \\ - \frac{\gamma_\pi^s(\alpha\gamma_\pi^s - (\alpha-\rho))}{(\alpha-\rho)\rho} (\psi_0 f_t + \psi_1 f_{t-1} + \dots + \psi_{r-1} f_{t-r+1}) \end{pmatrix} = 0 \quad (19)$$

$$\hat{E}\mathbf{x}_{2,t-\tau}(f_{t+1} - k - b_1 f_t - b_2 f_{t-1} - \dots - b_r f_{t-r+1}) = 0, \quad (20)$$

where $\mathbf{x}_{1,t} = (1 \ s_t)'$, $\mathbf{x}_{2,t} = (1 \ f_t)'$, and $\tau = 0, 1, \dots, p$ ^{31,32}.

3 Empirical Results

This section reports estimates of the persistence parameter α (or convergence rate parameter b) and their implied half-lives from aforementioned three estimation strategies.

We use CPIs and GDP deflators in order to construct real exchange rates with the US\$ as a base currency. I consider 19 industrialized countries³³ that provide 18 exchange rates. For interest rates, I use quarterly money market interest rates that are short-term interbank call rates rather than conventional short-term treasury bill rates, since we incorporate the Taylor Rule in the model where a central bank sets its target short-term market rate. For output deviations, we consider two different

³¹ p does not necessarily coincide with r .

³²In actual estimations, we use the aforementioned normalization again.

³³Among 23 industrialized countries classified by IMF, I dropped Greece, Iceland, and Ireland due to lack of reasonable number of observations. Luxembourg was not included because it has a currency union with Belgium.

measures of output gaps, quadratically detrended real GDP gap (see Clarida, Galí, and Gertler 1998)³⁴ and unemployment rate gaps (see Boivin 2005)³⁵. The data frequency is quarterly and from the IFS CD-ROM and DataStream³⁶. For GDP deflator-based real exchange rates, the observations span from 1979:III to 1998:IV for Eurozone countries, and from 1979:III to 2003:IV for Non-Eurozone EU countries and Non-EU countries with some exceptions due to lack of observations (see the note on Table 2 for complete description)³⁷. For CPI-based real exchange rates, the sample period is from 1979:III to 1998:IV for Eurozone countries, and from 1979:III to 2003:IV for the rest of the countries except Sweden (see the note on Table 3 for complete description).

The reason that our sample period starts from 1979:III is based on empirical evidence on the US Taylor Rule. As discussed in Section II, the inflation and exchange rate dynamics may greatly depend on the size of the central bank’s reaction coefficient to future inflation. We showed that the rationally expected future fundamental variables appear in the exchange rate and inflation dynamics only when the long-run inflation coefficient γ_π is strictly greater than unity. Clarida, Galí, and Gertler (1998, 2000) provide important empirical evidence for the existence of a structural break in the US Taylor Rule. Put it differently, they show that γ_π was strictly less than one during the pre-Volker era, while it became strictly greater than unity in the Volker-Greenspan era.

We implement similar GMM estimations for (4) as in Clarida, Galí, and Gertler (2000)³⁸³⁹ with longer sample period and report the results in Table 1 (see the note on Table 1 for detailed explanation). We use combinations of two different inflation measures and two output gap measures for three different sub-samples. Most coefficients were highly significant and specification tests by J -test

³⁴I also tried same analysis with the cyclical components of real GDP series from the HP-filter with 1600 of smoothing parameter. The results were quantitatively similar.

³⁵The unemployment gap is defined as a 5 year backward moving average subtracted by the current unemployment rate. This specification makes its sign consistent with that of the conventional output gap.

³⁶I obtained Danish and Japanese GDP deflator data from DataStream. IFS CD-ROM doesn’t provide reasonable number of observations for Danish deflator data. There was a seasonality problem in Japanese GDP deflator data in IFS CD-ROM until late 1979. All other data is from IFS CD-ROM.

³⁷In this paper, relevant Eurozone countries are Austria, Belgium, Finland, France, Germany, Italy, Netherlands, Portugal, and Spain. Non-Eurozone EU countries refer to Sweden, Denmark, and the UK. And Non-EU countries include Australia, Canada, Japan, New Zealand, Norway, Switzerland, and the US.

³⁸They used GDP deflator inflation along with the CBO output gaps (and HP detrended gaps).

³⁹Unlike them, we assume that the Fed targets current output gap rather than future deviations. However, this doesn’t make any significant changes to our results. And we include one lag of interest rate rather than two lags for simplicity.

were not rejected. More importantly, our requirement for the existence of a saddle path equilibrium met only for the Volker-Greenspan era. Therefore, we may conclude that this provides some empirical justification for the choice of our sample period.

>>>> **Insert Table 1 Here** <<<<<

Half-life estimates by the single equation (Section 2.3.1), the Hansen-Sargent method (Section 2.3.2), and finally the system method (Section 2.3.3) are reported in Table 2 for GDP deflator-based real exchange rates and in Table 3 for CPI-based real exchange rates. We implemented estimations using both gap measures, but report the full estimates with unemployment gaps along with the median half-life with quadratically detrended real GDP gaps (Median_x) in order to save space⁴⁰. We chose the degrees of autoregressive process of each fundamental term in (17) by the conventional BIC⁴¹. Standard errors were adjusted using the QS kernel estimator with automatic bandwidth selection in order to deal with unknown serial correlation problems⁴².

One interesting finding is that both the Hansen-Sargent method and the system method provide much shorter half-life estimates compared with ones from the single equation method (see Tables 2 and 3). The median half-life estimates from the single equation method were 3.06 and 2.59 years for GDP deflator-based and CPI-based real exchange rates, respectively. However, we obtained 1.49 and 1.37 years from the system method, and less than 1 year median half-life estimates from the Hansen-Sargent method⁴³. Interestingly, these estimates are roughly consistent with the average half-life estimates from the micro-data evidence by Crucini and Shintani (2004). For the OECD countries, their baseline half-life estimates for traded good prices were 1.0 years, while 1.1 and 1.6 years for all and non-traded good prices.

⁴⁰The results with quadratically detrended real GDP gaps were quantitatively similar.

⁴¹Chosen rs range from 2 to 4. We didn't consider the r greater than 4 in order to avoid the potential "many instrument" problem.

⁴²We impose the corresponding Taylor Rule coefficient GMM estimates (first stage estimation) throughout the whole estimation since joint estimation can be quite burdensome. We need to correct the standard errors by redefining the second step weighting matrix. However, since we acquired significant efficiency improvement over the single equation method, we believe such correction may not change the results much.

⁴³The results were quantitatively similar when the quadratically detrended real GDP series were used. See Median_x values for details.

Compared with Kim, Ogaki, and Yang (2003), our half-life estimates are longer than theirs. Using the system method with the restriction that was derived from the money demand function rather than the Taylor Rule, they implemented half-life estimations for CPI-based real exchange rates. Their half-life estimates range from 0.12 to 2.22 years, and their median half-life estimates for the full sample period was about 0.35 year⁴⁴, which is much shorter than our 1.37 year median half-life. However, our median half-life estimates are still shorter than the 3 to 5 year consensus half-life, and we believe that ours are reasonably short considering micro-data evidence⁴⁵. Furthermore, our results from the Hansen-Sargent method were more reliable than theirs as they obtained unreasonable estimates often.

Regarding efficiency, we obtained substantial efficiency gains from both the Hansen-Sargent method and the system method over the single equation method. As pointed out by Murray and Papell (2002), half-life estimates from single equation methods may provide virtually no useful information due to wide confidence intervals. Our half-life estimates from the single equation method were consistent with such a view. For most GDP deflator based real exchange rates with the exceptions of Australia, Denmark, and New Zealand and all CPI-based real exchange rates with the exception of Australia, standard errors for half-life estimates were very big. However, when we implement estimations by the Hansen-Sargent method and the system method, the standard errors were reduced significantly. Our results can be also considered as great improvement over Kim, Ogaki, and Yang (2003) who acquired only limited success in efficiency gains. We got even more dramatic improvement in efficiency for estimates by the Hansen-Sargent method, since their estimates by the Hansen-Sargent method produced extremely wide confidence intervals often.

Finally, we report likelihood ratio type tests and corresponding p -values. In the estimations by the system method, we impose a restriction that α in orthogonality conditions (18) and (19) are same. We can construct a likelihood ratio type test statistic from J -statistics for the unrestricted estimation and the estimation with such a restriction, which obeys χ^2 distribution asymptotically with degree of freedom 1. In most cases, our specification were not rejected.

⁴⁴They don't report the median estimate. Their median half-life estimate was obtained by our calculation.

⁴⁵For the survey on other micro-data evidence, see Crucini and Shintani (2004).

>>>> **Insert Table 2 Here** <<<<<

>>>> **Insert Table 3 Here** <<<<<

Based on our results shown here, we believe that the Hansen-Sargent method and the system method that incorporate a forward looking version Taylor Rule may greatly help resolving two aforementioned issues (from single equation approaches) in the PPP literature. Furthermore, it seems that deriving restrictions from the Taylor Rule rather than the money demand function was an appropriate choice based on our estimation results from each approach.

4 Conclusion

This paper has addressed two perennial issues of PPP literature, namely, unreasonably long half-life estimates of PPP deviations (Rogoff, 1996) and extremely wide confidence intervals of half-life point estimates (Murray and Papell, 2002). As a means of resolving these issues, Kim, Ogaki, and Yang (2003) have suggested using a system method utilizing economic theories or models in implementing estimations. Using post Bretton Woods CPI-based real exchange rates, they estimated much shorter half-lives (0.12 to 2.22 years) than the three- to five- year consensus half-life, but with only limited efficiency gains.

In contrast, we incorporated a forward-looking version of the Taylor Rule in the model, rather than the money demand function, in order to derive restrictions for half-life estimation. We obtained reasonably short median half-life estimates for both GDP deflator- and CPI-based real exchange rates for 19 developed countries, and these half-life estimates were shorter than the current consensus half-life estimates of three to five years. Interestingly, our median half-life estimates are roughly consistent with the micro-data evidence of Crucini and Shintani (2004).

With regard to efficiency, our half-life estimation using the Hansen-Sargent method and the system method greatly outperformed the single-equation method. In these cases, we found much smaller

standard errors for virtually all of our half-life estimates, as compared to those obtained using the single-equation method. Our use of the Taylor Rule, in place of the money demand function used by Kim, Ogaki, and Yang (2003), seems to have been an appropriate choice, given that we obtained substantial efficiency gains, even compared with their results.

Finally, our model has potentially important implications for exchange rate and inflation dynamics. In particular, we showed that rationally expected *future* fundamental variables enter these dynamics only when the long-run Taylor Rule inflation coefficient is greater than one. When it is less than one, exchange rate and inflation dynamics may be explained only by *past* fundamental variables and *any* martingale difference sequences.

A Derivation of (11)

Since $\mathbf{\Lambda}$ in (10) is diagonal, assuming $0 < \alpha < 1$ and $1 < \gamma_\pi < \frac{1}{1-\rho}$, we can solve the system as follows.

$$z_{1,t} = \sum_{j=0}^{\infty} \alpha^j h_{1,t-j-1} + \sum_{j=0}^{\infty} \alpha^j u_{t-j} \quad (\text{a1})$$

$$z_{2,t} = - \sum_{j=0}^{\infty} \left(\frac{1 - \gamma_\pi^s}{\rho} \right)^{j+1} E_t h_{2,t+j} \quad (\text{a2})$$

$$z_{3,t} = h_{3,t-1} + v_t, \quad (\text{a3})$$

where u_t and v_t are *any* martingale difference sequences.

Since $\mathbf{y}_t = \mathbf{V}\mathbf{z}_t$,

$$\begin{bmatrix} \Delta p_t \\ \Delta e_t \\ i_{t-1} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ \frac{\alpha\gamma_\pi^s}{\alpha-\rho} & 1 & 1 \\ \frac{\alpha\gamma_\pi^s}{\alpha-\rho} & 1 & 0 \end{bmatrix} \begin{bmatrix} z_{1,t} \\ z_{2,t} \\ z_{3,t} \end{bmatrix} \quad (\text{a4})$$

From first and second rows of (a4), we get the following.

$$\Delta e_t = \frac{\alpha\gamma_\pi^s}{\alpha-\rho} \Delta p_t - \frac{\alpha\gamma_\pi^s - (\alpha-\rho)}{\alpha-\rho} z_{2,t} - \frac{\alpha\gamma_\pi^s - (\alpha-\rho)}{\alpha-\rho} z_{3,t} \quad (\text{a5})$$

Now, we find the analytic solutions for \mathbf{z}_t . Since $\mathbf{h}_t = \mathbf{V}^{-1}\mathbf{c}_t$,

$$\mathbf{h}_t = \frac{1}{1 - \gamma_\pi^s} \begin{bmatrix} -\frac{\alpha-\rho}{\alpha\gamma_\pi^s - (\alpha-\rho)} & \frac{\alpha-\rho}{\alpha\gamma_\pi^s - (\alpha-\rho)} & 0 \\ \frac{\alpha\gamma_\pi^s}{\alpha\gamma_\pi^s - (\alpha-\rho)} & -\frac{\alpha\gamma_\pi^s}{\alpha\gamma_\pi^s - (\alpha-\rho)} & 1 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} E_t \Delta p_{t+1}^* - \alpha \Delta p_t^* + \eta_t + \iota + \gamma_x^s x_t - i_t^* \\ \gamma_\pi^s (E_t \Delta p_{t+1}^* - \alpha \Delta p_t^* + \eta_t) + \iota + \gamma_x^s x_t - i_t^* \\ \gamma_\pi^s (E_t \Delta p_{t+1}^* - \alpha \Delta p_t^* + \eta_t) + \iota + \gamma_x^s x_t - \gamma_\pi^s i_t^* \end{bmatrix},$$

and thus,

$$h_{1,t} = -\frac{\alpha-\rho}{\alpha\gamma_\pi^s - (\alpha-\rho)} (E_t \Delta p_{t+1}^* - \alpha \Delta p_t^* + \eta_t) \quad (\text{a6})$$

$$h_{2,t} = \frac{1}{1 - \gamma_\pi^s} \left[\frac{\rho\gamma_\pi^s}{\alpha\gamma_\pi^s - (\alpha-\rho)} (E_t \Delta p_{t+1}^* - \alpha \Delta p_t^* + \eta_t) + \iota + \gamma_x^s x_t - \gamma_\pi^s i_t^* \right] \quad (\text{a7})$$

$$h_{3,t} = -i_t^* \quad (\text{a8})$$

Plugging (a6) into (a1),

$$\begin{aligned} z_{1,t} &= -\frac{\alpha - \rho}{\alpha\gamma_\pi^s - (\alpha - \rho)} \sum_{j=0}^{\infty} \alpha^j (\Delta p_{t-j}^* - \alpha\Delta p_{t-j-1}^* + \eta_{t-j-1}) + \sum_{j=0}^{\infty} \alpha^j u_{t-j} \\ &= -\frac{\alpha - \rho}{\alpha\gamma_\pi^s - (\alpha - \rho)} \Delta p_t^* + \sum_{j=0}^{\infty} \alpha^j u_{t-j} - \frac{\alpha - \rho}{\alpha\gamma_\pi^s - (\alpha - \rho)} \sum_{j=0}^{\infty} \alpha^j \eta_{t-j-1} \end{aligned} \quad (\text{a9})$$

Plugging (a7) into (a2)⁴⁶,

$$\begin{aligned} z_{2,t} &= -\frac{\gamma_\pi^s}{\alpha\gamma_\pi^s - (\alpha - \rho)} \sum_{j=0}^{\infty} \left(\frac{1 - \gamma_\pi^s}{\rho}\right)^j (E_t \Delta p_{t+j+1}^* - \alpha E_t \Delta p_{t+j}^* + E_t \eta_{t+j}) \\ &\quad - \frac{1}{\rho} \sum_{j=0}^{\infty} \left(\frac{1 - \gamma_\pi^s}{\rho}\right)^j (\iota + \gamma_x^s E_t x_{t+j} - \gamma_\pi^s E_t i_{t+j}^*) \\ &= \frac{\alpha\gamma_\pi^s}{\alpha\gamma_\pi^s - (\alpha - \rho)} \Delta p_t^* - \frac{\gamma_\pi^s}{\alpha\gamma_\pi^s - (\alpha - \rho)} \eta_t - \frac{\iota}{\gamma_\pi^s - (1 - \rho)} \\ &\quad - \frac{\gamma_\pi^s}{\rho} \sum_{j=0}^{\infty} \left(\frac{1 - \gamma_\pi^s}{\rho}\right)^j E_t \Delta p_{t+j+1}^* - \frac{\gamma_\pi^s}{\rho} \sum_{j=0}^{\infty} \left(\frac{1 - \gamma_\pi^s}{\rho}\right)^j \left(\frac{\gamma_x^s}{\gamma_\pi^s} E_t x_{t+j} - E_t i_{t+j}^*\right) \end{aligned}$$

Then, denoting $E_t f_{t+j}$ as $-(E_t i_{t+j}^* - E_t \Delta p_{t+j+1}^*) + \frac{\gamma_x^s}{\gamma_\pi^s} E_t x_{t+j} = -E_t r_{t+j}^* + \frac{\gamma_x^s}{\gamma_\pi^s} E_t x_{t+j}$,

$$z_{2,t} = \frac{\alpha\gamma_\pi^s}{\alpha\gamma_\pi^s - (\alpha - \rho)} \Delta p_t^* - \frac{\gamma_\pi^s}{\alpha\gamma_\pi^s - (\alpha - \rho)} \eta_t - \frac{\iota}{\gamma_\pi^s - (1 - \rho)} - \frac{\gamma_\pi^s}{\rho} \sum_{j=0}^{\infty} \left(\frac{1 - \gamma_\pi^s}{\rho}\right)^j E_t f_{t+j} \quad (\text{a10})$$

Finally, plugging (a8) into (a3),

$$z_{3,t} = -i_{t-1}^* + v_t \quad (\text{a11})$$

⁴⁶We use the fact $E_t \eta_{t+j} = 0$, $j = 1, 2, \dots$.

Now, plugging (a10) and (a11) into (a5),

$$\begin{aligned} \Delta e_t = & \frac{\alpha\gamma_\pi^s}{\alpha-\rho}\Delta p_t - \frac{\alpha\gamma_\pi^s}{\alpha-\rho}\Delta p_t^* + \frac{\gamma_\pi^s}{\alpha-\rho}\eta_t + \frac{\alpha\gamma_\pi^s - (\alpha-\rho)}{(\alpha-\rho)(\gamma_\pi^s - (1-\rho))}v_t \\ & + \frac{\gamma_\pi^s(\alpha\gamma_\pi^s - (\alpha-\rho))}{(\alpha-\rho)\rho} \sum_{j=0}^{\infty} \left(\frac{1-\gamma_\pi^s}{\rho}\right)^j E_t f_{t+j} + \frac{\alpha\gamma_\pi^s - (\alpha-\rho)}{\alpha-\rho} i_{t-1}^* - \frac{\alpha\gamma_\pi^s - (\alpha-\rho)}{\alpha-\rho} v_t \end{aligned} \quad (\text{a12})$$

Updating (a12) once and applying law of iterated expectations,

$$\begin{aligned} \Delta e_{t+1} = & \hat{i} + \frac{\alpha\gamma_\pi^s}{\alpha-\rho}\Delta p_{t+1} - \frac{\alpha\gamma_\pi^s}{\alpha-\rho}\Delta p_{t+1}^* + \frac{\alpha\gamma_\pi^s - (\alpha-\rho)}{\alpha-\rho} i_t^* \\ & + \frac{\gamma_\pi^s(\alpha\gamma_\pi^s - (\alpha-\rho))}{(\alpha-\rho)\rho} \sum_{j=0}^{\infty} \left(\frac{1-\gamma_\pi^s}{\rho}\right)^j E_t f_{t+j+1} + \omega_{t+1}, \end{aligned} \quad (\text{a13})$$

where

$$\hat{i} = \frac{\alpha\gamma_\pi^s - (\alpha-\rho)}{(\alpha-\rho)(\gamma_\pi^s - (1-\rho))}v_t,$$

$$\begin{aligned} \omega_{t+1} = & \frac{\gamma_\pi^s(\alpha\gamma_\pi^s - (\alpha-\rho))}{(\alpha-\rho)\rho} \sum_{j=0}^{\infty} \left(\frac{1-\gamma_\pi^s}{\rho}\right)^j (E_{t+1} f_{t+j+1} - E_t f_{t+j+1}) \\ & + \frac{\gamma_\pi^s}{\alpha-\rho}\eta_{t+1} - \frac{\alpha\gamma_\pi^s - (\alpha-\rho)}{\alpha-\rho} v_{t+1}, \end{aligned}$$

and,

$$E_t \omega_{t+1} = 0$$

References

- BOIVIN, J. (2005): “Has US Monetary Policy Changed? Evidence from Drifting Coefficients and Real Time Data,” NBER Working Paper No. 11314.
- CASSEL, G. (1921): *The World’s Monetary Problems*. E.P. Dutton and Co., New York.
- CHEN, S.-S., AND C. ENGEL (2005): “Does ”Aggregation Bias” Explain the PPP Puzzle?,” *Pacific Economic Review*, 10, 49–72.
- CLARIDA, R., J. GALI, AND M. GERTLER (1998): “Monetary Policy Rules in Practice: Some International Evidence,” *European Economic Review*, 42, 1033–1067.
- (1999): “The Science of Monetary Policy: A New Keynesian Perspective,” *Journal of Economic Literature*, 37, 1661–1707.
- (2000): “Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory,” *Quarterly Journal of Economics*, 115, 147–180.
- CRUCINI, M. J., AND M. SHINTANI (2004): “Persistence in Law-of-One-Price Deviations: Evidence from Micro-Data,” Working Paper, Vanderbilt University.
- DEVEREUX, M. B., AND C. ENGEL (2005): “Expectations and Exchange Rate Policy,” manuscript.
- ENGEL, C., AND K. D. WEST (2002): “Taylor Rules and the Deutschmark-Dollar Exchange Rate,” manuscript, Department of Economics, University of Wisconsin.
- (2003): “Exchange Rates and Fundamentals,” manuscript, Department of Economics, University of Wisconsin.
- FRANKEL, J., AND A. K. ROSE (1996): “A Panel Project on Purchasing Power Parity: Mean Reversion Within and Between Countries,” *Journal of International Economics*, 40, 209–224.
- FRENKEL, J. A. (1976): “A Monetary Approach to the Exchange Rate: Doctrinal Aspects and Empirical Evidence,” *Scandinavian Journal of Economics*, 78, 200–224.

- HANSEN, L. P., AND T. J. SARGENT (1980): "Formulating and Estimating Dynamic Linear Rational Expectations Models," *Journal of Economic Dynamics and Control*, 2, 7–46.
- (1981): "A Note on Wiener-Kolmogorov Prediction Formulas for Rational Expectations Models," Research Department Staff Report 69, Federal Reserve Bank of Minneapolis, Minneapolis, MN.
- (1982): "Instrumental Variables Procedures for Estimating Linear Rational Expectations Models," *Journal of Monetary Economics*, 9, 263–296.
- IMBS, J., H. MUMTAZ, M. RAVN, AND H. REY (2005): "PPP Strikes Back: Aggregatopm Amd the Real Exchange Rate," *Quarterly Journal of Economics*, 120, 1–44.
- ISARD, P. (1977): "How Far Can We Push the Law of One Price?," *American Economic Review*, 67, 942–948.
- JUDD, J. F., AND G. D. RUDEBUSCH (1998): "Taylor's Rule and the Fed: 1970 - 1997," *Federal Reserve Bank of San Francisco Economics Review*, 3, 3–16.
- KIM, H. (2005a): "Half Life Estimation under the Taylor Rule: Two Goods Model," manuscript, Department of Economics, Ohio State University.
- (2005b): "Purchasing Power Parity under a Taylor Rule Type Monetary Policy," manuscript, Department of Economics, Ohio State University.
- KIM, J., AND M. OGAKI (2004): "Purchasing Power Parity for Traded and Non-Traded Goods: A Structural Error Correction Model Approach," *Monetary and Economic Studies*, 22, 1–26.
- KIM, J., M. OGAKI, AND M.-S. YANG (2003): "Structural Error Correction Models: Instrumental Variables Methods and an Application to an Exchange Rate Model," the Rochester Center for Economic Research Working Paper No. 502, University of Rochester.
- LOPEZ, C., C. MURRAY, AND D. H. PAPELL (2005): "State of the Art Unit Root Tests and Purchasing Power Parity," *Journal of Money, Credit, and Banking*, 37, 361–369.

- LUCAS, ROBERT E., J. (1982): “Interest Rates and Currency Prices in a Two Country World,” *Journal of Monetary Economics*, 10, 335–359.
- MARK, N. C. (2001): *International Macroeconomics and Finance: Theory and Econometric Methods*. Blackwell Publishers Inc., Oxford, UK.
- (2005): “Changing Monetary Policy Rules, Learning, and Real Exchange Rate Dynamics,” National Bureau of Economic Research Working Paper No. 11061.
- MURRAY, C. J., AND D. H. PABELL (2002): “The Purchasing Power Parity Paradigm,” *Journal of International Economics*, 56, 1–19.
- (2005): “Do Panels Help Solve the Purchasing Power Parity Puzzle?,” *Journal of Business and Economic Statistics*, forthcoming.
- MUSSA, M. (1982): “A Model of Exchange Rate Dynamics,” *Journal of Political Economy*, 90, 74–104.
- OBSTFELD, M. (2004): “Pricing-to-Market, the Interest-Rate Rule, and the Exchange Rate,” manuscript, Department of Economics, University of California, Berkeley.
- OBSTFELD, M., AND K. ROGOFF (1995): “Exchange Rate Dynamic Redux,” *Journal of Political Economy*, 103, 624–660.
- OBSTFELDT, M., AND K. ROGOFF (1999): *Foundations of International Macroeconomics*. The MIT Press, Cambridge, Massachusetts.
- ORPHANIDES, A. (2001): “Monetary Policy Rules, Macroeconomic Stability, and Inflation: A View from the Trenches,” European Central Bank Working Paper No. 115.
- PABELL, D. H. (1997): “Searching for Stationarity: Purchasing Power Parity Under the Current Float,” *Journal of International Economics*, 43, 313–332.
- PARSLEY, D. C., AND S.-J. WEI (2004): “A Prism Into the PPP Puzzles: The Micro-Foundations of Big Mac Real Exchange Rates,” manuscript.

- ROGOFF, K. (1996): “The Purchasing Power Parity Puzzle,” *Journal of Economic Literature*, 34, 647–668.
- SARGENT, T. J. (1987): *Macroeconomic Theory*. Academic Press, New York, second edn.
- TAYLOR, A. M. (2002): “A Century of Purchasing Power Parity,” *Review of Economics and Statistics*, 84, 139–150.
- TAYLOR, J. B. (1993): “Discretion versus Policy Rules in Practice,” *Carnegie-Rochester Series on Public Policy*, 39, 195–214.
- (1999a): “A Historical Analysis of Monetary Policy Rules,” in *Monetary Policy Rules*, ed. by J. B. Taylor, Chicago. University of Chicago Press.
- (1999b): *Monetary Policy Rules*. University of Chicago Press, Chicago.
- WEST, K. D. (1989): “Estimation of Linear Rational Expectations Models, In the Presence of Deterministic Terms,” *Journal of Monetary Economics*, 24, 437–442.
- WU, Y. (1996): “Are Real Exchange Rates Nonstationary? Evidence from a Panel Data Test,” *Journal of Money, Credit, and Banking*, 28, 54–63.

Table 1. GMM Estimation of the US Taylor Rule

Inflation	Deviation	Sample Period	γ_π (s.e.)	γ_x (s.e.)	ρ (s.e.)	J (pv)
GDP Deflator	Real GDP	1960.Q1~1979.Q2	0.998 (0.033)	0.670 (0.023)	0.417 (0.014)	14.418 (0.851)
		1979.Q3~1998.Q4	2.529 (0.068)	0.234 (0.086)	0.632 (0.017)	16.292 (0.753)
		1979.Q3~2003.Q4	2.900 (0.128)	0.216 (0.096)	0.686 (0.025)	22.191 (0.386)
	Unemployment	1960.Q1~1979.Q2	0.920 (0.015)	0.191 (0.008)	0.213 (0.024)	12.647 (0.921)
		1979.Q3~1998.Q4	3.311 (0.240)	0.222 (0.055)	0.744 (0.034)	14.998 (0.823)
		1979.Q3~2003.Q4	3.532 (0.304)	0.255 (0.055)	0.773 (0.036)	16.904 (0.717)
CPI	Real GDP	1960.Q1~1979.Q2	0.573 (0.058)	1.529 (0.232)	0.723 (0.037)	16.604 (0.735)
		1979.Q3~1998.Q4	2.903 (0.166)	0.084 (0.058)	0.742 (0.021)	15.949 (0.773)
		1979.Q3~2003.Q4	2.935 (0.237)	0.144 (0.112)	0.751 (0.027)	19.144 (0.576)
	Unemployment	1960.Q1~1979.Q2	0.944 (0.061)	0.255 (0.041)	0.594 (0.045)	17.284 (0.694)
		1979.Q3~1998.Q4	2.941 (0.153)	0.125 (0.038)	0.806 (0.012)	16.334 (0.751)
		1979.Q3~2003.Q4	2.539 (0.143)	0.105 (0.069)	0.793 (0.027)	18.414 (0.623)

Notes: i) Inflations are quarterly changes in log price level ($\ln p_t - \ln p_{t-1}$). ii) Quadratically detrended real GDP series are used for real GDP output deviations. iii) Unemployment gaps are 5 year backward moving average unemployment rates minus current unemployment rates. iv) The set of instruments includes four lags of federal funds rate, inflation, output deviation, long-short interest rate spread, commodity price inflation, and M2 growth rate. v) Standard errors were adjusted using the QS kernel with automatic bandwidth selection, and are reported in parentheses. vi) J in last column refers to J -test statistics, and corresponding p -values are in parentheses.

Table 2. Half-Life Estimation: GDP Deflator-Based Real Exchange Rates

Country	System Method		Hansen-Sargent		Single Equation Method		LR (<i>pv</i>)
	Half Life (<i>s.e.</i>)	α (<i>s.e.</i>)	Half Life (<i>s.e.</i>)	α (<i>s.e.</i>)	Half Life (<i>s.e.</i>)	α (<i>s.e.</i>)	
Australia	1.735 (0.521)	0.905 (0.027)	0.842 (0.658)	0.814 (0.131)	2.603 (1.251)	0.936 (0.030)	1.735 (0.188)
Austria	2.839 (1.494)	0.941 (0.030)	0.808 (0.185)	0.807 (0.040)	3.909 (3.515)	0.957 (0.038)	1.931 (0.165)
Belgium	1.703 (0.732)	0.903 (0.040)	0.524 (0.098)	0.718 (0.044)	3.332 (2.786)	0.949 (0.041)	1.345 (0.246)
Canada	3.209 (1.633)	0.947 (0.026)	0.976 (0.385)	0.837 (0.059)	4.833 (4.223)	0.965 (0.030)	1.264 (0.261)
Denmark	0.761 (0.089)	0.796 (0.021)	0.762 (0.077)	0.797 (0.018)	1.076 (0.398)	0.851 (0.051)	0.397 (0.529)
Finland	1.368 (0.257)	0.881 (0.021)	0.956 (0.155)	0.834 (0.024)	3.393 (2.650)	0.950 (0.038)	0.346 (0.556)
France	1.875 (1.166)	0.912 (0.052)	0.515 (0.075)	0.714 (0.035)	2.788 (2.171)	0.940 (0.045)	0.338 (0.561)
Germany	2.249 (1.354)	0.926 (0.043)	0.698 (0.100)	0.780 (0.028)	3.589 (3.335)	0.953 (0.043)	1.945 (0.163)
Italy	0.719 (0.060)	0.786 (0.016)	0.694 (0.055)	0.779 (0.015)	4.217 (4.300)	0.960 (0.040)	0.968 (0.325)
Japan	1.614 (0.494)	0.898 (0.030)	0.649 (0.038)	0.766 (0.012)	3.897 (2.889)	0.957 (0.032)	7.996 (0.005)
Netherlands	1.664 (1.056)	0.901 (0.060)	0.902 (0.561)	0.825 (0.099)	1.876 (1.238)	0.912 (0.056)	0.097 (0.755)
New Zealand	0.786 (0.110)	0.802 (0.025)	0.638 (0.045)	0.762 (0.015)	2.032 (0.948)	0.918 (0.037)	4.030 (0.045)
Norway	0.837 (0.142)	0.813 (0.028)	0.813 (0.142)	0.808 (0.030)	1.213 (0.630)	0.867 (0.064)	0.156 (0.693)
Portugal	0.802 (0.023)	0.806 (0.005)	0.795 (0.062)	0.804 (0.014)	13.47 (28.94)	0.987 (0.027)	1.546 (0.214)
Spain	0.533 (0.021)	0.722 (0.009)	0.566 (0.036)	0.736 (0.014)	3.851 (2.983)	0.956 (0.033)	0.182 (0.670)
Sweden	0.525 (0.061)	0.719 (0.028)	0.435 (0.064)	0.671 (0.039)	1.890 (1.498)	0.912 (0.066)	2.008 (0.156)
Switzerland	0.705 (0.064)	0.782 (0.018)	0.617 (0.049)	0.755 (0.017)	2.795 (2.011)	0.940 (0.042)	2.686 (0.101)
UK	1.626 (0.673)	0.899 (0.040)	0.677 (0.066)	0.774 (0.019)	2.524 (1.967)	0.934 (0.050)	3.197 (0.074)
Median (Avg)	1.491 (1.419)	0.890	0.696 (0.715)	0.780	3.064 (3.516)	0.945	-
Median _x (Avg)	1.726 (1.865)	0.905	0.711 (0.805)	0.784	-	-	-

Notes: i) The US\$ is the base currency. ii) Unemployment gaps are used for output deviations. iii) Median_x is the median when quadratically detrended output gaps are used. iv) Chosen parameters are $(\gamma_\pi, \gamma_x, \rho) = (3.311, 0.222, 0.744)$ for countries where observations end in 1998.IV, while $(3.532, 0.255, 0.773)$ for countries where observations end in 2003.IV. v) Sample periods are 1979.III ~ 1998.IV for most Eurozone countries (Austria, Finland, France, Germany, Italy, Netherlands, Portugal, Spain), and 1979.III~2003.IV for one Non-Eurozone EU countries (UK) and most Non-EU countries (Australia, Canada, Japan, Norway, Switzerland, US). Among Eurozone countries, Belgium's sample spans from 1980.I to 1998.IV. Among Non-Eurozone EU countries, Denmark's observations span from 1984.IV to 2003.IV, and Sweden's sample period is from 1980.I to 2001.I. Among Non-EU countries, New Zealand's sample period starts from 1982.II until 2003.IV. vi) Standard errors were adjusted using the QS kernel with automatic bandwidth selection, and are reported in parentheses. vii) LR in last column refers to the likelihood ratio type test statistics and corresponding *p*-values are in parentheses.

Table 3. Half-Life Estimation: CPI-Based Real Exchange Rates

Country	System Method		Hansen-Sargent		Single Equation Method		LR (<i>pv</i>)
	Half Life (<i>s.e.</i>)	α (<i>s.e.</i>)	Half Life (<i>s.e.</i>)	α (<i>s.e.</i>)	Half Life (<i>s.e.</i>)	α (<i>s.e.</i>)	
Australia	1.422 (0.282)	0.885 (0.021)	0.707 (0.063)	0.783 (0.017)	2.572 (1.188)	0.935 (0.029)	2.015 (0.156)
Austria	0.732 (0.073)	0.789 (0.019)	0.624 (0.057)	0.758 (0.019)	2.608 (1.798)	0.936 (0.043)	4.265 (0.039)
Belgium	0.679 (0.066)	0.775 (0.019)	0.604 (0.070)	0.750 (0.025)	2.038 (1.096)	0.918 (0.042)	3.501 (0.061)
Canada	8.406 (7.994)	0.980 (0.019)	1.797 (2.778)	0.908 (0.135)	5.970 (5.448)	0.971 (0.026)	0.055 (0.815)
Denmark	1.929 (0.838)	0.914 (0.036)	0.648 (0.203)	0.765 (0.064)	2.351 (1.272)	0.929 (0.037)	2.131 (0.144)
Finland	4.264 (4.322)	0.960 (0.040)	1.284 (0.771)	0.874 (0.071)	3.051 (2.235)	0.945 (0.039)	0.736 (0.391)
France	1.323 (0.537)	0.877 (0.047)	0.484 (0.127)	0.699 (0.066)	2.015 (1.241)	0.918 (0.049)	0.304 (0.581)
Germany	0.844 (0.124)	0.814 (0.025)	0.717 (0.072)	0.874 (0.071)	1.841 (1.136)	0.910 (0.053)	0.400 (0.527)
Italy	1.414 (0.481)	0.885 (0.037)	1.156 (0.483)	0.861 (0.054)	2.607 (2.011)	0.936 (0.048)	0.499 (0.527)
Japan	0.575 (0.056)	0.740 (0.022)	0.526 (0.073)	0.719 (0.033)	3.188 (2.182)	0.947 (0.035)	5.946 (0.015)
Netherlands	0.754 (0.124)	0.795 (0.030)	0.735 (0.125)	0.790 (0.032)	1.688 (1.003)	0.902 (0.055)	1.656 (0.198)
New Zealand	2.240 (0.598)	0.926 (0.019)	0.613 (0.078)	0.754 (0.027)	3.142 (0.984)	0.946 (0.016)	0.743 (0.389)
Norway	1.636 (0.676)	0.899 (0.039)	0.336 (0.117)	0.597 (0.107)	2.142 (1.268)	0.922 (0.044)	4.329 (0.037)
Portugal	0.932 (0.078)	0.830 (0.013)	0.525 (0.047)	0.719 (0.021)	5.538 (5.930)	0.969 (0.032)	1.410 (0.235)
Spain	4.218 (4.236)	0.960 (0.040)	0.561 (0.301)	0.734 (0.122)	3.704 (3.109)	0.954 (0.037)	1.996 (0.158)
Sweden	4.949 (6.607)	0.966 (0.045)	0.931 (0.163)	0.830 (0.027)	5.773 (7.641)	0.970 (0.039)	0.904 (0.342)
Switzerland	1.169 (0.404)	0.862 (0.044)	0.480 (0.098)	0.697 (0.051)	1.976 (1.108)	0.916 (0.045)	0.160 (0.689)
UK	0.579 (0.108)	0.741 (0.041)	0.428 (0.131)	0.667 (0.083)	1.796 (1.170)	0.908 (0.057)	8.139 (0.004)
Median (Avg)	1.369 (2.115)	0.881	0.619 (0.731)	0.756	2.590 (3.000)	0.936	-
Median _x (Avg)	1.408 (1.923)	0.859	0.504 (0.558)	0.712	-	-	-

Notes: i) The US\$ is the base currency. ii) Unemployment gaps are used for output deviations. iii) Median_x is the median when quadratically detrended output gaps are used. iv) Chosen parameters are $(\gamma_\pi, \gamma_x, \rho) = (2.941, 0.125, 0.806)$ for countries where observations end in 1998.IV, while $(2.539, 0.105, 0.793)$ for countries where observations end in 2003.IV. v) Sample periods are 1979.III ~ 1998.IV for Eurozone countries (Austria, Belgium, Finland, France, Germany, Italy, Netherlands, Portugal, Spain), and 1979.III ~ 2003.IV for Non-Eurozone EU countries (Denmark, UK) except Sweden, and Non-EU countries (Australia, Canada, Japan, New Zealand, Norway, Switzerland, US). Sweden's observations span from 1979.III to 2001.IV. vi) Standard errors were adjusted using the QS kernel with automatic bandwidth selection, and are reported in parentheses. vii) LR in last column refers to the likelihood ratio type test statistics and corresponding *p*-values are in parentheses.