

Half-Life Estimation under the Taylor Rule: Two Goods Model

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Abstract

This paper estimates and compares half-lives of Purchasing Power Parity (PPP) deviations for real exchange rates in the Post-Bretton Woods era. We employ a GMM system method that incorporates a forward looking version Taylor Rule. In a similar framework, Kim (2005) reported much shorter half-life point estimates for the non-service goods prices than those for service goods prices, though with quite wide confidence intervals. However, we find that half-life estimates are about the same irrespective of the choice of price indexes, which is consistent with the studies that report only moderate or no difference (e.g., Wu 1996, Murray and Papell 2002). More importantly, we obtained a substantial improvement in efficiency, which is enough to make statistically meaningful comparisons between the size of half-life estimates. Our model also shows the real exchange rate dynamics may be quite different depending on the central bank's pattern of systematic response to inflation.

Keywords: Purchasing Power Parity, Taylor Rule, Half-Life of PPP Deviations

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1 Introduction

Purchasing Power Parity (PPP, henceforth) has been one of the most useful building blocks for many influential exchange rate models (e.g., Frenkel 1976, Lucas 1982, Obstfeld and Rogoff 1995). The main logic of PPP is that if goods market arbitrage can ensure broad parity over a sufficiently wide range of individual products, then the overall price indices should also exhibit similar parity across countries.

In presence of nominal rigidities, financial factors such as monetary shocks may cause temporal deviations of the real exchange rate away from its equilibrium PPP level. If PPP holds in the long-run, however, such deviations will die out eventually, since nominal shocks would be neutral in the long-run.

Accepting PPP as a valid long-run proposition¹, Kim (2005) estimated half-lives of PPP deviations, the time required for a deviation to halfway adjust to its long-run equilibrium level, in a system method framework developed by Kim et al (2007), which combines economic theories with the univariate model of real exchange rates. He obtained much shorter half-life point estimates for the non-service consumption deflators than those for the service consumption deflators^{2,3}, though with quite wide confidence intervals. Using the same framework, Kim and Ogaki (2004) obtained similar evidence for producer price index (PPI)- and consumer price index (CPI)-based real exchange rates⁴.

However, such results are in contrast to many other researches that found only moderate or no significant difference in half-life point estimates. Wu (1996), for example, obtained around 2.1- and 2.5-year half-life estimates for wholesale price index (WPI)- and CPI-based real exchange rates, respectively. Murray and Papell (2002) also found that their half-life point estimates were consistent with Rogoff (1996)'s three- to five-year consensus half life no matter what price indexes

¹Empirical evidence on the validity of long-run PPP is mixed. See Rogoff (1996) for a survey on this issue.

²His median half-life point estimates were 0.67 and 3.36 years for non-service and service consumption deflators, respectively. The medians were calculated from his Tables 2 and 4.

³Following Stockman and Tesar (1995), he used the non-service consumption deflator as the traded good price, while the non-traded good price was measured by the service consumption deflator. See Kim (2005) for details.

⁴Both Kim (2005) and Kim and Ogaki (2004) also estimated the half-lives for the general price index using a single good approach described in Kim, Ogaki, and Yang (2007).

are used to construct real exchange rates⁵. Interestingly, Wei and Parsley (1995) reported four- to five-year half-lives even when they used 12 traded good sector deflators.

This paper constitutes a modification of the work of Kim (2005) and Kim and Ogaki (2004), who combined the univariate model of real exchange rates with the conventional money market equilibrium condition, which they identified using a money demand function. However, the money demand function can be quite unstable especially in the short-run. So the use of the money demand function may not have been ideal. Instead, this paper constructs a system of the exchange rate and inflation that incorporates a forward looking version of the Taylor Rule monetary policy, where the central bank sets its target interest rate based on the expected future changes in the general price index such as the GDP deflator⁶.

This paper utilizes two sets of exchange rates in the Post-Bretton Woods era. First, we consider 19 industrialized countries that provide 18 current float real exchange rates based on the PPIs and CPIs. Second, we use the Kim (2005)'s data set for 7 developed countries. We obtain about 3-year median half-life estimates for all real exchange rates we consider in this paper. More importantly, we obtain a substantial improvement in efficiency over the results of Kim (2005) and Kim and Ogaki (2004) as well as the conventional univariate estimation approaches⁷. Interestingly, our *grid-t confidence intervals* (Hansen, 1999) were consistent with Rogoff (1996)'s three- to five-year consensus half-life *point* estimate. It seems that our efficiency gains partly come from the choice of the Taylor rule rather than using the money demand function, and partly due to better choice of instruments than those of Kim (2005) and Kim and Ogaki (2004). With such efficiency gains, it is possible to make statistically meaningful comparisons between the sizes of half-life estimates.

It should be noted that, however, we are not arguing that convergence rates for the traded good prices are about the same as those of the non-traded goods prices. Instead, we report some empirical evidence against the use of some of the popular aggregate prices such as CPIs for the

⁵They used the current float CPI-based real exchange rates and the long-horizon WPI-based real exchange rates, which extends Lee (1976)'s data.

⁶For a model where the central bank targets the CPI inflation, see Kim (2007).

⁷Murray and Papell (2002) argue that univariate methods provide virtually no information regarding the size of the half-lives due to extremely wide confidence intervals.

non-tradables price index.

The rest of this paper is organized as follows. In section 2, we construct a system of stochastic difference equations for the exchange rate and inflation explicitly incorporating a forward looking version of the Taylor Rule with interest rate smoothing policy. Then, we discuss our GMM estimation strategy for the estimable equations that are derived from the system. In section 3, a description of the data and the estimation results are provided. Section 4 concludes.

2 Model Specification and Estimation Strategy

In this section, we assume that there are two different baskets of products in each country. Let p_t^1 and p_t^2 be the logs of the price indexes of those baskets in the home country. We further assume that the log of the general price index (p_t^G) is a weighted average of p_t^1 and p_t^2 . That is,

$$p_t^G = \delta p_t^1 + (1 - \delta)p_t^2, \quad (1)$$

where δ is a strictly positive weight parameter. Foreign price indices are similarly defined. One natural interpretation of the equation (1) is that the GDP deflator is a geometric weighted average of the PPI and CPI, where the GDP basket includes the CPI and PPI baskets⁸.

2.1 Gradual Adjustment Equation

Our model starts with a simple univariate stochastic process of real exchange rates. Let p_t be the log of the domestic price level (p_t^1 or p_t^2), p_t^* be the log of the foreign price level (p_t^{1*} or p_t^{2*}), and e_t be the log of the nominal exchange rate as the unit price of the foreign currency in terms of the domestic currency. And the log of the real exchange rate, s_t , is defined as $p_t^* + e_t - p_t$.

Here, we simply assume that PPP holds in the long-run without any formal econometric test⁹. Put it differently, we assume that there exists a cointegrating vector $[1 - 1 - 1]'$ for a vector $[p_t \ p_t^* \ e_t]'$,

⁸Engel (1999), Kakkar and Ogaki (1999), and Betts and Kehoe (2005) also employ this assumption.

⁹The empirical evidence on the validity of PPP in the long-run is mixed. It should be noted, however, that even when we have some evidence against PPP, such outcomes may have resulted from lack of power of existing unit root tests in small samples, and are subject to observational equivalence problems.

where p_t , p_t^* , and e_t are all difference stationary processes. Under this assumption, real exchange rates can be represented as the following stationary univariate autoregressive process of degree one.

$$s_{t+1} = d + \alpha s_t + \varepsilon_{t+1}, \quad (2)$$

where α is a strictly positive persistence parameter that is less than one¹⁰.

Kim et al (2007) show that the equation (2) could be implied by the following error correction model of real exchange rates by Mussa (1982) with a *known* cointegrating relation described earlier.

$$\Delta p_{t+1} = b [\mu - (p_t - p_t^* - e_t)] + E_t \Delta p_{t+1}^* + E_t \Delta e_{t+1}, \quad (3)$$

where $\mu = E(p_t - p_t^* - e_t)$, $b = 1 - \alpha$, $d = -(1 - \alpha)\mu$, $\varepsilon_{t+1} = \varepsilon_{1t+1} + \varepsilon_{2t+1} = (E_t \Delta e_{t+1} - \Delta e_{t+1}) + (E_t \Delta p_{t+1}^* - \Delta p_{t+1}^*)$, and $E_t \varepsilon_{t+1} = 0$. $E(\cdot)$ denotes the unconditional expectation operator, and $E_t(\cdot)$ is the conditional expectation operator on I_t , the economic agent's information set at time t .

One interpretation of (3) is that p_t adjusts instantaneously to the expected *change* in its PPP, while it adjusts to its unconditional PPP level, $E(p_t^* + e_t)$ only slowly with the constant convergence rate $b (= 1 - \alpha)$, which is a strictly positive constant less than one by construction.

2.2 Taylor Rule Model

We assume that the central bank in the home country continuously sets its optimal target interest rate (i^T) by the following forward looking version of the Taylor Rule.

$$i_t^T = \tilde{l} + \gamma_\pi E_t \Delta p_{t+1}^G + \gamma_x x_t, \quad (4)$$

¹⁰This is the so-called Dickey-Fuller estimation model. It is also possible to estimate half-lives by the augmented Dickey-Fuller estimation model in order to avoid a serial correlation problem in ε_{t+1} . However, as shown in Murray and Papell (2002), these two methodologies provide roughly similar half-life estimates. So it seems that our AR(1) specification is not a bad approximation.

where \tilde{i} is a constant that includes a certain long-run equilibrium real interest rate along with a target inflation rate¹¹, and γ_π and γ_x are the *long-run* Taylor Rule coefficients on expected future inflation ($E_t\Delta p_{t+1}^G$) and current output deviations (x_t)¹², respectively. Note that we are implicitly assuming that the central bank is targeting general inflation, which is defined as changes in a general price index such as the GDP deflator.

We further assume that the central bank attempts to smooth the current actual interest rate (i_t) by the following rule.

$$i_t = (1 - \rho)i_t^T + \rho i_{t-1} \quad (5)$$

That is, the current actual interest rate is a weighted average of the target interest rate and the previous period's interest rate, where ρ is the interest rate smoothing parameter. Plugging (4) into (5), we derive the following forward looking version of the Taylor Rule equation with the interest rate smoothing policy.

$$i_t = (1 - \rho)\tilde{i} + (1 - \rho)\gamma_\pi E_t\Delta p_{t+1}^G + (1 - \rho)\gamma_x x_t + \rho i_{t-1} \quad (6)$$

Next, we assume that uncovered interest parity holds. That is,

$$E_t\Delta e_{t+1} = i_t - i_t^*, \quad (7)$$

where i_t^* is the foreign interest rate.

Without loss of generality, we assume that equations (2) or (3) hold for p_t^1 and p_t^{1*} without imposing any restriction on p_t^2 and p_t^{2*} . That is, PPP is assumed to hold in the long-run for the prices of first baskets of goods¹³. With this assumption, we rewrite (3) as the following equation

¹¹See Clarida, Galí, and Gertler (1998, 2000) for details.

¹²If we assume that the central bank responds to expected future output deviations rather than current deviations, we can simply modify the model by replacing x_t with $E_t x_{t+1}$. However, this does not make any significant difference to our results.

¹³This does not mean that PPP doesn't hold for the second baskets of goods.

in level variables.

$$p_{t+1}^1 = b\mu + E_t e_{t+1} + (1-b)p_t^1 - (1-b)e_t + E_t p_{t+1}^{1*} - (1-b)p_t^{1*} \quad (8)$$

Taking differences and rearranging it, (8) can be rewritten as follows.

$$\Delta p_{t+1}^1 = E_t \Delta e_{t+1} + \alpha \Delta p_t^1 - \alpha \Delta e_t + [E_t \Delta p_{t+1}^{1*} - \alpha \Delta p_t^{1*} + \eta_t], \quad (9)$$

where $\alpha = 1 - b$ and $\eta_t = \eta_{1,t} + \eta_{2,t} = (e_t - E_{t-1}e_t) + (p_t^{T*} - E_{t-1}p_t^{T*})$.

Since we are interested in the dynamic behavior of Δp_{t+1}^1 rather than Δp_{t+1}^G , we rewrite (6) in terms of Δp_{t+1}^1 using (1). That is,

$$i_t = \iota + \delta \gamma_\pi^s E_t \Delta p_{t+1}^1 + \gamma_x^s x_t + \rho i_{t-1} + (1-\delta) \gamma_\pi^s E_t \Delta p_{t+1}^2, \quad (10)$$

where $\iota = (1-\rho)\tilde{\iota}$, and $\gamma_\pi^s = (1-\rho)\gamma_\pi$ and $\gamma_x^s = (1-\rho)\gamma_x$ are *short-run* Taylor Rule coefficients. Combining (7) and (10), we obtain the following.

$$E_t \Delta e_{t+1} = \iota + \delta \gamma_\pi^s E_t \Delta p_{t+1}^1 + \gamma_x^s x_t + \rho i_{t-1} + (1-\delta) \gamma_\pi^s E_t \Delta p_{t+1}^2 - i_t^* \quad (11)$$

Denoting $\gamma_\pi^{s1} = \delta \gamma_\pi^s$ and $\gamma_\pi^{s2} = (1-\delta) \gamma_\pi^s$, we construct the following system of stochastic difference equations from (9), (10) and (11).

$$\begin{bmatrix} 1 & -1 & 0 \\ -\gamma_\pi^{s1} & 1 & 0 \\ -\gamma_\pi^{s1} & 0 & 1 \end{bmatrix} \begin{bmatrix} E_t \Delta p_{t+1}^1 \\ E_t \Delta e_{t+1} \\ i_t \end{bmatrix} = \begin{bmatrix} \alpha & -\alpha & 0 \\ 0 & 0 & \rho \\ 0 & 0 & \rho \end{bmatrix} \begin{bmatrix} \Delta p_t^1 \\ \Delta e_t \\ i_{t-1} \end{bmatrix} + \begin{bmatrix} E_t \Delta p_{t+1}^{1*} - \alpha \Delta p_t^{1*} + \eta_t \\ \iota + \gamma_\pi^{s2} E_t \Delta p_{t+1}^2 + \gamma_x^s x_t - i_t^* \\ \iota + \gamma_\pi^{s2} E_t \Delta p_{t+1}^2 + \gamma_x^s x_t \end{bmatrix} \quad (12)$$

For notational simplicity, let's rewrite (12) as the following equation in matrix.

$$\mathbf{A} E_t \mathbf{y}_{t+1} = \mathbf{B} \mathbf{y}_t + \mathbf{x}_t, \quad (13)$$

Note that \mathbf{A} is nonsingular, and thus has a well-defined inverse. Thus, (13) can be rewritten as follows.

$$\begin{aligned} E_t \mathbf{y}_{t+1} &= \mathbf{A}^{-1} \mathbf{B} \mathbf{y}_t + \mathbf{A}^{-1} \mathbf{x}_t \\ &= \mathbf{D} \mathbf{y}_t + \mathbf{c}_t, \end{aligned} \tag{14}$$

where $\mathbf{D} = \mathbf{A}^{-1} \mathbf{B}$, $\mathbf{c}_t = \mathbf{A}^{-1} \mathbf{x}_t$. By the eigenvalue decomposition, (14) can be expressed as

$$E_t \mathbf{y}_{t+1} = \mathbf{V} \mathbf{\Lambda} \mathbf{V}^{-1} \mathbf{y}_t + \mathbf{c}_t, \tag{15}$$

where $\mathbf{D} = \mathbf{V} \mathbf{\Lambda} \mathbf{V}^{-1}$ and

$$\mathbf{V} = \begin{bmatrix} 1 & 1 & 1 \\ \frac{\alpha \gamma_\pi^{s1}}{\alpha - \rho} & 1 & 1 \\ \frac{\alpha \gamma_\pi^{s1}}{\alpha - \rho} & 1 & 0 \end{bmatrix}, \quad \mathbf{\Lambda} = \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \frac{\rho}{1 - \gamma_\pi^{s1}} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Pre-multiplying (15) by \mathbf{V}^{-1} ,

$$\mathbf{E}_t \mathbf{z}_{t+1} = \mathbf{\Lambda} \mathbf{z}_t + \mathbf{h}_t, \tag{16}$$

where $\mathbf{z}_t = \mathbf{V}^{-1} \mathbf{y}_t$ and $\mathbf{h}_t = \mathbf{V}^{-1} \mathbf{c}_t$.

Note that, among non-zero eigenvalues in $\mathbf{\Lambda}$, α is between 0 and 1 by definition, while $\frac{\rho}{1 - \gamma_\pi^{s1}}$ ($= \frac{\rho}{1 - \delta(1 - \rho)\gamma_\pi}$) is greater than unity as long as $1 < \delta\gamma_\pi < \frac{1}{1 - \rho}$. Therefore, if the domestic central bank's systematic response to inflation is aggressive enough ($\delta\gamma_\pi > 1$)¹⁴, the system of stochastic difference equations (15) will have a saddle path equilibrium, in which rationally expected future fundamental variables appear in the exchange rate and inflation dynamics. Otherwise, the system would follow a purely backward looking solution path to be completely determined only by *past* fundamental variables and *any* martingale difference sequences.

Under the assumption of $\delta\gamma_\pi > 0$, it turns out that the solution to (15) satisfies the following

¹⁴The condition $\delta\gamma_\pi < \frac{1}{1 - \rho}$ is easily met for all sample periods we consider in this paper.

relation (see Appendix for its derivation).

$$\begin{aligned}\Delta e_{t+1} &= \bar{v} + \frac{\alpha\gamma_\pi^{s1}}{\alpha-\rho}\Delta p_{t+1}^1 - \frac{\alpha\gamma_\pi^{s1}}{\alpha-\rho}\Delta p_{t+1}^{1*} + \frac{\alpha\gamma_\pi^{s1} - (\alpha-\rho)}{\alpha-\rho}i_t^* \\ &\quad + \frac{\gamma_\pi^{s1}(\alpha\gamma_\pi^{s1} - (\alpha-\rho))}{\rho(\alpha-\rho)} \sum_{j=0}^{\infty} \left(\frac{1-\gamma_\pi^{s1}}{\rho}\right)^j E_t f_{t+j+1} + \omega_{t+1},\end{aligned}\tag{17}$$

where,

$$\bar{v} = \frac{\alpha\gamma_\pi^{s1} - (\alpha-\rho)}{(\alpha-\rho)(\gamma_\pi^{s1} - (1-\rho))}v_t,$$

$$\begin{aligned}E_t f_{t+j} &= -(E_t i_{t+j}^* - E_t \Delta p_{t+j+1}^{1*}) + \frac{1-\delta}{\delta} E_t \Delta p_{t+j+1}^2 + \frac{\gamma_x}{\delta\gamma_\pi} E_t x_{t+j} \\ &= -E_t r_{t+j}^* + \frac{1-\delta}{\delta} E_t \Delta p_{t+j+1}^2 + \frac{\gamma_x}{\delta\gamma_\pi} E_t x_{t+j},\end{aligned}$$

$$\begin{aligned}\omega_{t+1} &= \frac{\gamma_\pi^{s1}(\alpha\gamma_\pi^{s1} - (\alpha-\rho))}{\rho(\alpha-\rho)} \sum_{j=0}^{\infty} \left(\frac{1-\gamma_\pi^{s1}}{\rho}\right)^j (E_{t+1} f_{t+j+1} - E_t f_{t+j+1}) \\ &\quad + \frac{\gamma_\pi^{s1}}{\alpha-\rho} \eta_{t+1} - \frac{\alpha\gamma_\pi^{s1} - (\alpha-\rho)}{\alpha-\rho} v_{t+1},\end{aligned}$$

and,

$$E_t \omega_{t+1} = 0$$

Or, (17) can be rewritten with full parameter specification as follows.

$$\begin{aligned}\Delta e_{t+1} &= \bar{v} + \frac{\alpha\delta(1-\rho)\gamma_\pi}{\alpha-\rho}\Delta p_{t+1}^1 - \frac{\alpha\delta(1-\rho)\gamma_\pi}{\alpha-\rho}\Delta p_{t+1}^{1*} + \frac{\alpha\delta(1-\rho)\gamma_\pi - (\alpha-\rho)}{\alpha-\rho}i_t^* \\ &\quad + \frac{\delta(1-\rho)\gamma_\pi(\alpha\delta(1-\rho)\gamma_\pi - (\alpha-\rho))}{\rho(\alpha-\rho)} \sum_{j=0}^{\infty} \left(\frac{1-\delta(1-\rho)\gamma_\pi}{\rho}\right)^j E_t f_{t+j+1} + \omega_{t+1},\end{aligned}\tag{18}$$

Note that one of the key variables f_t is a proxy that summarizes the rationally expected *future* fundamental variables such as foreign real interest rates (r_t^*) and domestic output deviations.

It is important to realize that if $\delta\gamma_\pi$ is strictly less than unity, the restriction implied by (18)

may not be valid, since the system would have a purely backward looking equilibrium rather than a saddle path equilibrium¹⁵. However, if we believe that the central bank has been reacting to inflation aggressively enough (e.g., post-Volker era), rationally expected future Taylor Rule fundamentals should be explicitly considered in half-life estimation procedures. In a nutshell, real exchange rate dynamics critically depends on the size of γ_π given δ . As will be shown later, we will provide some supporting evidence on such a requirement for the existence of a saddle path equilibrium for the sample period we consider¹⁶.

2.3 Estimation Strategy

2.3.1 Univariate Equation Method

First approach is the conventional univariate equation method that is based on the equations (2) or (3). Assuming that PPP holds for p_t^1 and p_t^{1*} , the equation (3) can be transformed to the following estimable equation.

$$\Delta p_{t+1}^1 = b [\mu - (p_t^1 - p_t^{1*} - e_t)] + \Delta p_{t+1}^{1*} + \Delta e_{t+1} + \varepsilon_{t+1}, \quad (19)$$

where $\varepsilon_{t+1} = \varepsilon_{1t+1} + \varepsilon_{2t+1} = (E_t \Delta e_{t+1} - \Delta e_{t+1}) + (E_t \Delta p_{t+1}^{1*} - \Delta p_{t+1}^{1*})$ and $E_t \varepsilon_{t+1} = 0$. It is straightforward to show that (19) implies (2).

Note that the convergence parameter b or the persistence parameter $\alpha (= 1 - b)$ can be consistently estimated by the conventional least squares method under the maintained cointegration assumption¹⁷ as long as there's no measurement error¹⁸. Once we get the point estimate of α , the half-life of the real exchange rate can be obtained by $\frac{\ln(.5)}{\ln \alpha}$, and standard errors can be calculated

¹⁵In that case, the conventional vector autoregressive (VAR) estimation method may apply.

¹⁶In contrast to Clarida, Gali, and Gertler (1998, 2000), Taylor (1999a), and Judd and Rudebusch (1998), Orphanides (2001) found the estimates of the inflation coefficient γ_π to be consistently greater than one in both the pre- and post-Volker regimes. In any case, our model specification is still valid.

¹⁷It is well-known that the standard errors obtained from the least squares or parametric bootstrap methods may not be valid asymptotically in presence of serially correlated errors. It would be necessary to use either Newey-West or QS kernel covariance estimators, or to use the moving block bootstrap method.

¹⁸If there is a measurement error problem, α may not be even consistent, since the aforementioned cointegrating relation assumption may not hold. We can deal with this problem by a two stage cointegration method that directly estimates the cointegrating vector as a first stage estimation. See Kim et al (2007) for details.

by the delta method.

2.3.2 GMM System Method

Our second estimation strategy deals with the equations (17) or (18). It should be noted that (17) has an infinite sum of rationally expected discounted future fundamental variables, which complicates our estimation procedure. Following Hansen and Sargent (1980, 1982), we linearly project $E_t(\cdot)$ onto Ω_t , the econometrician's information set at time t , which is a subset of I_t . Denoting $\hat{E}_t(\cdot)$ as such a linear projection operator onto Ω_t , we rewrite (17) as follows.

$$\begin{aligned} \Delta e_{t+1} = & \bar{i} + \frac{\alpha\gamma_\pi^{s1}}{\alpha - \rho}\Delta p_{t+1}^1 - \frac{\alpha\gamma_\pi^{s1}}{\alpha - \rho}\Delta p_{t+1}^{1*} + \frac{\alpha\gamma_\pi^{s1} - (\alpha - \rho)}{\alpha - \rho}i_t^* \\ & + \frac{\gamma_\pi^{s1}(\alpha\gamma_\pi^{s1} - (\alpha - \rho))}{\rho(\alpha - \rho)} \sum_{j=0}^{\infty} \left(\frac{1 - \gamma_\pi^{s1}}{\rho}\right)^j \hat{E}_t f_{t+j+1} + \xi_{t+1}, \end{aligned} \quad (20)$$

where

$$\xi_{t+1} = \omega_{t+1} + \frac{\gamma_\pi^{s1}(\alpha\gamma_\pi^{s1} - (\alpha - \rho))}{\rho(\alpha - \rho)} \sum_{j=0}^{\infty} \left(\frac{1 - \gamma_\pi^{s1}}{\rho}\right)^j \left(E_t f_{t+j+1} - \hat{E}_t f_{t+j+1}\right),$$

and

$$\hat{E}_t \xi_{t+1} = 0$$

by the law of iterated projections.

For the instrumental variables that are included in Ω_t , Kim (2005) chooses Δp_{t-2}^* and Δp_{t-4}^* , which seems to be rather unusual, because those variables may not be enough to fully explain the money demand fundamental variables. Instead, we suggest $\Omega_t = \{f_t, f_{t-1}, f_{t-2}, \dots\}$, that is, we assume that the econometricians possess full information about the past history of the Taylor rule fundamentals. This assumption would be an innocent one under the stationarity assumption for f_t , and it can greatly lessen the burden of our GMM estimations by significantly reducing the number of coefficients to be estimated.

For this purpose, we assume that f_t be a zero mean covariance stationary (for now), linearly

indeterministic stochastic process so that it has the following Wold representation.

$$f_t = c(L)\nu_t, \quad (21)$$

where $\nu_t = f_t - \hat{E}_{t-1}f_t$ and $c(L) = 1 + c_1L + c_2L^2 + \dots$ is square summable. Assuming the invertibility of $c(L)$, (21) can be rewritten as the following autoregressive representation.

$$d(L)f_t = \nu_t, \quad (22)$$

where $d(L) = c^{-1}(L) = 1 - d_1L - d_2L^2 - \dots$. It can be shown that the the following relation should hold when we linearly project $\sum_{j=0}^{\infty} \left(\frac{1-\gamma s^1}{\rho}\right)^j E_t f_{t+j+1}$ onto Ω_t ¹⁹.

$$\sum_{j=0}^{\infty} \delta^j \hat{E}_t f_{t+j+1} = \psi(L)f_t = \left[\frac{1 - (\delta^{-1}d(\delta))^{-1} d(L)L^{-1}}{1 - (\delta^{-1}L)^{-1}} \right] f_t, \quad (23)$$

where $\delta = \frac{1-\gamma s^1}{\rho}$.

For actual estimation, we assume that f_t can be represented by a finite order AR(r) process²⁰, that is, $d(L) = 1 - \sum_{j=1}^r d_j L^j$, where $r < \infty$. Then, it can be shown that the coefficients of $\psi(L)$ can be computed recursively as follows²¹.

$$\psi_0 = (1 - \delta d_1 - \dots - \delta^r d_r)^{-1}$$

$$\psi_r = 0$$

$$\psi_{j-1} = \delta\psi_j + \delta\psi_0 d_j,$$

where $j = 1, 2, \dots, r$. Then, the GMM estimation of (17) can be implemented by simultaneously

¹⁹See Hansen and Sargent (1980) for details.

²⁰We can use conventional Akaike Information criteria (AIC) or Bayesian Information criteria (BIC) in order to choose the degree of such autoregressive processes.

²¹See Sargent (1987) for details.

estimating next two equations²².

$$\begin{aligned} \Delta e_{t+1} = & \bar{e} + \frac{\alpha\gamma_\pi^{s1}}{\alpha-\rho}\Delta p_{t+1}^1 - \frac{\alpha\gamma_\pi^{s1}}{\alpha-\rho}\Delta p_{t+1}^{1*} + \frac{\alpha\gamma_\pi^{s1} - (\alpha-\rho)}{\alpha-\rho}i_t^* \\ & + \frac{\gamma_\pi^{s1}(\alpha\gamma_\pi^{s1} - (\alpha-\rho))}{\rho(\alpha-\rho)}(\psi_0 f_t + \psi_1 f_{t-1} + \cdots + \psi_{r-1} f_{t-r+1}) + \xi_{t+1}, \end{aligned} \quad (24)$$

and

$$f_{t+1} = k + d_1 f_t + d_2 f_{t-1} + \cdots + d_r f_{t-r+1} + \nu_{t+1}, \quad (25)$$

where k is a constant, and $\hat{E}_t \nu_{t+1} = 0$.

Recall that we assume a zero-mean covariance stationary process for f_t following Hansen and Sargent (1980). If the variable of interest has a non-zero unconditional mean, we can either demean it prior to the estimation or include a constant but leave its coefficient unconstrained. West (1989) showed that the further efficiency gain can be obtained by imposing additional restrictions on the deterministic term. However, the imposition of such an additional restriction is quite burdensome, so we simply add a constant here.

Finally, we combine equations (24) and (25) with (2) or (19) as Kim (2005) and Kim and Ogaki (2004). That is, we implement a GMM estimation for the three equations (2), (24), and (25) simultaneously. Note that, in the system method, we restrict α in (24) and (2) are the same. In so doing, we may be able to acquire further efficiency gains by using more information.

A GMM estimation, then, can be implemented by the following $2(p+2)$ orthogonality conditions.

$$E\mathbf{x}_{1,t}(s_{t+1} - d - \alpha s_t) = 0 \quad (26)$$

$$\hat{E}\mathbf{x}_{2,t-\tau} \left(\begin{array}{l} \Delta e_{t+1} - \bar{e} - \frac{\alpha\delta(1-\rho)\gamma_\pi}{\alpha-\rho}\Delta p_{t+1}^1 + \frac{\alpha\delta(1-\rho)\gamma_\pi}{\alpha-\rho}\Delta p_{t+1}^{1*} - \frac{\alpha\delta(1-\rho)\gamma_\pi - (\alpha-\rho)}{\alpha-\rho}i_t^* \\ - \frac{\delta(1-\rho)\gamma_\pi(\alpha\delta(1-\rho)\gamma_\pi - (\alpha-\rho))}{\rho(\alpha-\rho)}(\psi_0 f_t + \psi_1 f_{t-1} + \cdots + \psi_{r-1} f_{t-r+1}) \end{array} \right) = 0 \quad (27)$$

$$\hat{E}\mathbf{x}_{2,t-\tau}(f_{t+1} - k - d_1 f_t - d_2 f_{t-1} - \cdots - d_r f_{t-r+1}) = 0, \quad (28)$$

²²In actual estimations, we normalized (24) by multiplying $(\alpha-\rho)$ to each side in order to reduce nonlinearity.

where $\mathbf{x}_{1,t} = (1 \ s_t)'$, $\mathbf{x}_{2,t} = (1 \ f_t)'$, and $\tau = 0, 1, \dots, p$ ²³.

3 Empirical Results

In this section, we report estimates of the persistence parameter α (or convergence rate parameter b) and their implied half-lives from aforementioned two estimation strategies for the two data sets.

First, we use conventional PPIs and CPIs in order to construct real exchange rates with the US dollar as a base currency. We consider 19 industrialized countries²⁴ that provide 18 real exchange rates. For interest rates, we use quarterly money market interest rates that are short-term inter-bank call rates rather than using the conventional short-term treasury bill rates. This is because we incorporate the Taylor Rule in the model where the central bank sets its target short-term market rate based on changes in the GDP deflator and output deviations. For output deviations, we use quadratically detrended real GDPs (Clarida, Galí, and Gertler 1998)²⁵. The data frequency is quarterly and from IFS CD-ROM. The observations span from 1973:I to 1998:IV for most Eurozone countries, and from 1973:I to 2003:IV for Non-Eurozone EU countries and Non-EU countries with some exceptions due to lack of observations (see the notes on Table 2 and 3 for complete description)²⁶.

Before we proceed for half-life estimations, we need to check if the estimation equations are well-specified. As discussed in Section II, the dynamic system of the exchange rate and inflation may differ greatly, depending on the systematic response pattern of the central bank to future inflation. That is, we showed that the rationally expected future Taylor Rule fundamentals appear in the real exchange rate dynamics only when the long-run inflation coefficient is big enough ($\delta\gamma_\pi > 1$).

²³Note that p does not necessarily coincide with r .

²⁴For CPI-based real exchange rates, among 23 industrialized countries classified by IMF, I dropped Greece, Iceland, and Ireland due to lack of reasonable number of observations. Luxembourg was not included because it has a currency union with Belgium. For PPI-based real exchange rates, I dropped Portugal also again due to lack of observations.

²⁵We also tried same analysis with the cyclical components of real GDP series from the HP-filter with 1600 of smoothing parameter as well as the Congressional Budget Office (CBO) output gaps. The results were quantitatively similar.

²⁶In this paper, relevant Eurozone countries are Austria, Belgium, Finland, France, Germany, Italy, Netherlands, Portugal, and Spain. Non-Eurozone EU countries refer to Sweden, Denmark, and the UK. And Non-EU countries include Australia, Canada, Japan, New Zealand, Norway, Switzerland, and the US.

Otherwise, the orthogonality conditions (27) and (28) would become invalid and other estimation strategies such as the VAR method should be employed.

Clarida, Galí, and Gertler (1998, 2000) provide important empirical evidence for the existence of a structural break in the US Taylor Rule. They show that the US central bank has reacted to the future inflation more aggressively during the post-Volker era (1979:III \sim) than the pre-Volker era. However, Orphanides (2001) reports empirical evidence that the Fed has consistently maintained aggressive policy stance to inflationary pressure.

Following Clarida, Galí, and Gertler (2000)²⁷²⁸, we implemented a similar GMM estimation for the US Taylor Rule using longer sample period. We use the quarterly changes in the GDP deflator as a general inflation measure, and quadratically detrended real GDPs for output gap. The results are reported in Table 1 (see the note on Table 1 for detailed explanation). Most coefficients were highly significant with the exception of the long-run output deviations coefficients for the Post-Bretton Woods era, and the J -test didn't reject any of our model specification (not reported). More importantly, our requirement for the existence of a saddle path equilibrium met only for the Post-Bretton Woods era and for the Post-Volker era²⁹. Therefore, we may conclude that this provides some empirical justification for our model specifications.

>>>>> **Insert Table 1 Here** <<<<<<

We report our GMM system method estimates for the persistence parameter estimates and the corresponding half-life estimates in Tables 2 (CPI-based) and 3 (PPI-based) for the Post-Bretton Woods era³⁰. For comparisons, we also report univariate estimation results in Table 8. Our major

²⁷They used GDP deflator inflation along with the CBO output gaps (and HP detrended gaps).

²⁸Unlike them, we assume that the Fed targets current output gap rather than future deviations. However, this doesn't make any significant changes to our results. And we include one lag of interest rate rather than two lags for simplicity.

²⁹There is no obvious way of how to measure δ , the weight parameter on the first basket of goods. Since we treat the first basket as a so-called tradable goods where PPI holds without imposing any restriction on the second basket of goods, I measured δ as the ratio of the sum of durable and non-durable consumption expenditures to the total expenditures, treating service expenditure as a proxy for non-tradables. From the long-horizon US consumption data (1959:I \sim 2003:IV), I got 0.43.

³⁰We chose 8 lags of the Taylor rule fundamentals as the instruments, because the BIC rule chose 8 lags for the majority real exchange rates.

findings are as follows. First, unlike Kim and Ogaki (2004)³¹, our GMM point estimates are about the same irrespective of the choice of prices, and our point estimates are similar as those of univariate regression estimates.

Second, our standard errors are much smaller for both real exchange rates, while Kim and Ogaki (2004) reported relatively smaller standard errors for PPI-based real exchange rates than CPI-based real exchange rates. It is well-known that when a variable exhibits high persistence, asymptotic normality may not be obtained in small samples so that statistical inferences based on standard errors can be misleading. In order to avoid such problems, we construct and report grid- t confidence intervals by Hansen (1999). Interestingly, our grid- t *confidence intervals* are consistent with the Rogoff (1996)'s 3- to 5-year consensus half-life *point* estimate, while those of univariate regressions and Kim and Ogaki (2004, not reported) are consistent with Murray and Papell (2002)'s median-unbiased confidence intervals of which the upper bounds are typically infinity.

>>>>> **Insert Table 2 Here** <<<<<<

>>>>> **Insert Table 3 Here** <<<<<<

Our second data set is the one used by Kim (2005) who used the non-service consumption deflator as the traded good price, while the non-traded good price was measured by the service consumption deflator following Stockman and Tesar (1995). Our GMM estimates are reported in Table 4. Using the money demand function, Kim (2005) obtained 0.67- and 3.36-year median half-life estimates for non-service and service consumption deflators, respectively³²³³. However, we obtained 2.75- and 3.15-year median half-life estimates using the same data. Our point estimates are consistent with the Rogoff's consensus half-life again irrespective of the choice of prices. We

³¹Their median half-life estimates were 0.28 and 1.08 years for PPI- and CPI-based real exchange rates.

³²His point estimates for non-service consumption deflators range from 0.13 to 2.71 years, and from 0.30 to 15.52 years for service consumption deflators.

³³The medians were calculated from his Tables 2 and 4.

also obtain a substantial improvement in efficiency over Kim (2005)'s work. Unlike his work, we obtained much tighter confidence intervals for both real exchange rate series.

>>>>> **Insert Table 4 Here** <<<<<

We also implement GMM estimations for the Post-Volker era that starts from the third quarter of 1979. The results are reported in Tables 5 to 7. The main drawback of using this sub-sample is that the estimates may suffer from small-sample bias. We obtain qualitatively similar results as those of the Post-Bretton Woods era in the sense that the half-life point estimates are about the same and corresponding confidence intervals are compact. However, the point estimates are substantially smaller when we use the Post-Volker era data. The point estimates are around 1-year, and confidence intervals never exceeded 2-year. It is possible for such discrepancies are resulted from small-sample bias. However, it may be partly due to the fact we drop the sample period of the great depreciation episode (1973 ~ 1979) for dollar-based exchange rates. Interestingly, Papell (2002) finds much stronger evidence for PPP when he considers structure breaks for the great depreciations (1973 ~ 1979 and 1985 ~ 1987) and the great appreciation (1980 ~ 1984).

>>>>> **Insert Table 5 Here** <<<<<

>>>>> **Insert Table 6 Here** <<<<<

>>>>> **Insert Table 7 Here** <<<<<

It is very interesting that we obtained both qualitatively and quantitatively different results compared with those of Kim (2005) and Kim and Ogaki (2004), since we use similar estimation techniques. One potentially serious problem in their estimation results is that their point estimates

are associated with quite big standard errors. Especially for CPIs or service consumption deflators, it wasn't possible to make a statistically meaningful reasoning for the size of the half-life due to extremely wide confidence intervals. Even though they obtained relatively better estimates (in terms of efficiency) for PPIs and non-service consumption deflators, the standard errors were still quite big for many estimates.

However, we obtained a substantial improvement in efficiency for every real exchange rate we consider here. It should be noted that our results greatly outperform the univariate equation approach too. Murray and Papell (2002) point out that the univariate equation approach provides virtually no useful information regarding the size of the half-lives due to extremely wide confidence intervals of the point estimates. Such wide confidence intervals are also found when we correct the small sample bias using Hansen (1999)'s grid- t method (see Tables 8 and 9). Interestingly, our grid- t confidence intervals for our system method estimates are much tighter so that the statistically meaningful comparisons are possible.

>>>> **Insert Table 8 Here** <<<<<

>>>> **Insert Table 9 Here** <<<<<

Therefore, we may conclude that even though Kim (2004) and Kim and Ogaki (2002) found significant difference in half-lives, one may not fully accept the results as statistically meaningful ones. Our results, however, provide more efficient half-life estimates for all real exchange rates and our finding that the half-lives are about the same irrespective of the choice of aggregate price indexes should be considered to be statistically reliable. It should be noted that, though, we are not claiming that the traded goods prices and the non-traded goods prices share similar convergence rates. We provide some empirical evidence against the use of some of the popular measure of non-traded goods prices such as CPIs and service goods consumption deflators.

Finally, the natural question is that why this paper produces completely different results from those of Kim (2005) and Kim and Ogaki (2007) even though the estimation strategies are very similar. It should be noted that one of the advantages of using the system method is that it may provide more efficient estimates as long as the imposed restrictions are valid. Rather than using the money demand function as they do, this paper incorporates a forward looking version of the Taylor Rule. Therefore, it seems that deriving restrictions from the Taylor Rule was an appropriate choice. Another possibility is the difference in the choice of instrumental variables. They use two- and four-lag foreign inflations Δp_{t-2}^* and Δp_{t-4}^* for instrumental variables. However, this seems to be rather unusual and those variables may not fully explain the money demand fundamentals. Instead, we assume that the econometrician's information set includes the history of the Taylor rule fundamentals. We believe that this choice is intuitively more appealing and such a method greatly reduces computational burden as well.

4 Conclusion

In this paper, we employed a system method framework that explicitly incorporates a forward looking version of the Taylor Rule to estimate and compare half-lives of PPP deviations for various type of real exchange rates. In a similar framework, Kim (2005) obtained much shorter half-life point estimates for the non-service consumption deflators than those for the service consumption deflators, which is in contrast to other studies that report moderate or no difference (e.g., Wu 1996, Murray and Papell 2002). Kim and Ogaki (2004) report similar evidence for CPI- and PPI-based real exchange rates.

However, their point estimates were associated with quite wide confidence intervals, and thus provide statistically less useful information regarding the size of half-life point estimates. In contrast, we obtain a substantial improvement in efficiency over the work of Kim (2005) and Kim and Ogaki (2004). Our half-life estimations produce much tighter confidence intervals irrespective of the choice of prices, enabling statistically meaningful comparisons of the length of half-life point estimates. Thus, our conclusion that the half-lives are about the same irrespective of the choice

of aggregate prices may be statistically more reliable one. And we believe that the use of the Taylor Rule as well as the appropriate choice of instrumental variables greatly help obtaining such a substantial efficiency gain.

A Derivation of (17)

Note that $\mathbf{\Lambda}$ in (16) is a diagonal matrix. Therefore, assuming $0 < \alpha < 1$ and $1 < \delta\gamma_\pi < \frac{1}{1-\rho}$, we can solve the system of stochastic difference equation (16) as the following three equations.

$$z_{1,t} = \sum_{j=0}^{\infty} \alpha^j h_{1,t-j-1} + \sum_{j=0}^{\infty} \alpha^j u_{t-j} \quad (\text{a1})$$

$$z_{2,t} = - \sum_{j=0}^{\infty} \left(\frac{1 - \gamma_\pi^{s1}}{\rho} \right)^{j+1} E_t h_{2,t+j} \quad (\text{a2})$$

$$z_{3,t} = h_{3,t-1} + v_t, \quad (\text{a3})$$

where u_t and v_t are any martingale difference sequences.

Since $\mathbf{y}_t = \mathbf{V}\mathbf{z}_t$,

$$\begin{bmatrix} \Delta p_t^1 \\ \Delta e_t \\ i_{t-1} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ \frac{\alpha\gamma_\pi^{s1}}{\alpha-\rho} & 1 & 1 \\ \frac{\alpha\gamma_\pi^{s1}}{\alpha-\rho} & 1 & 0 \end{bmatrix} \begin{bmatrix} z_{1,t} \\ z_{2,t} \\ z_{3,t} \end{bmatrix} \quad (\text{a4})$$

Multiplying the first row of (a4) by $\frac{\alpha\gamma_\pi^{s1}}{\alpha-\rho}$ and subtract it from the second row, we get the following.

$$\Delta e_t = \frac{\alpha\gamma_\pi^{s1}}{\alpha-\rho} \Delta p_t^1 - \frac{\alpha\gamma_\pi^{s1} - (\alpha-\rho)}{\alpha-\rho} z_{2,t} - \frac{\alpha\gamma_\pi^{s1} - (\alpha-\rho)}{\alpha-\rho} z_{3,t} \quad (\text{a5})$$

Now, we need to find the analytic solutions for \mathbf{z}_t . Since $\mathbf{h}_t = \mathbf{V}^{-1}\mathbf{c}_t$,

$$\mathbf{h}_t = \frac{1}{1 - \gamma_\pi^{s1}} \begin{bmatrix} -\frac{\alpha-\rho}{\alpha\gamma_\pi^{s1} - (\alpha-\rho)} & \frac{\alpha-\rho}{\alpha\gamma_\pi^{s1} - (\alpha-\rho)} & 0 \\ \frac{\alpha\gamma_\pi^{s1}}{\alpha\gamma_\pi^{s1} - (\alpha-\rho)} & -\frac{\alpha\gamma_\pi^{s1}}{\alpha\gamma_\pi^{s1} - (\alpha-\rho)} & 1 \\ 0 & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} E_t \Delta p_{t+1}^{1*} - \alpha \Delta p_t^{1*} + \eta_t + \iota + \gamma_\pi^{s2} E_t \Delta p_{t+1}^2 + \gamma_x^s x_t - i_t^* \\ \gamma_\pi^{s1} (E_t \Delta p_{t+1}^{1*} - \alpha \Delta p_t^{1*} + \eta_t) + \iota + \gamma_\pi^{s2} E_t \Delta p_{t+1}^2 + \gamma_x^s x_t - i_t^* \\ \gamma_\pi^{s1} (E_t \Delta p_{t+1}^{1*} - \alpha \Delta p_t^{1*} + \eta_t) + \iota + \gamma_\pi^{s2} E_t \Delta p_{t+1}^2 + \gamma_x^s x_t - \gamma_\pi^{s1} i_t^* \end{bmatrix},$$

and thus,

$$h_{1,t} = -\frac{\alpha - \rho}{\alpha\gamma_\pi^{s1} - (\alpha - \rho)} (E_t\Delta p_{t+1}^{1*} - \alpha\Delta p_t^{1*} + \eta_t) \quad (\text{a6})$$

$$h_{2,t} = \frac{1}{1 - \gamma_\pi^{s1}} \left[\frac{\rho\gamma_\pi^{s1}}{\alpha\gamma_\pi^{s1} - (\alpha - \rho)} (E_t\Delta p_{t+1}^{1*} - \alpha\Delta p_t^{1*} + \eta_t) + \iota + \gamma_\pi^{s2} E_t\Delta p_{t+1}^2 + \gamma_x^s x_t - \gamma_\pi^{s1} i_t^* \right] \quad (\text{a7})$$

$$h_{3,t} = -i_t^* \quad (\text{a8})$$

Plugging (a6) into (a1), we get,

$$\begin{aligned} z_{1,t} &= -\frac{\alpha - \rho}{\alpha\gamma_\pi^{s1} - (\alpha - \rho)} \sum_{j=0}^{\infty} \alpha^j (\Delta p_{t-j}^{1*} - \alpha\Delta p_{t-j-1}^{1*} + \eta_{t-j-1}) + \sum_{j=0}^{\infty} \alpha^j u_{t-j} \quad (\text{a9}) \\ &= -\frac{\alpha - \rho}{\alpha\gamma_\pi^{s1} - (\alpha - \rho)} \Delta p_t^{1*} - \frac{\alpha - \rho}{\alpha\gamma_\pi^{s1} - (\alpha - \rho)} \sum_{j=0}^{\infty} \alpha^j \eta_{t-j-1} + \sum_{j=0}^{\infty} \alpha^j u_{t-j} \end{aligned}$$

Plugging (a7) into (a2),

$$\begin{aligned} z_{2,t} &= -\frac{\gamma_\pi^{s1}}{\alpha\gamma_\pi^{s1} - (\alpha - \rho)} \sum_{j=0}^{\infty} \left(\frac{1 - \gamma_\pi^{s1}}{\rho} \right)^j (E_t\Delta p_{t+j+1}^{1*} - \alpha E_t\Delta p_{t+j}^{1*} + E_t\eta_{t+j}) \quad (\text{a10}) \\ &\quad - \frac{1}{\rho} \sum_{j=0}^{\infty} \left(\frac{1 - \gamma_\pi^{s1}}{\rho} \right)^j (\iota + \gamma_\pi^{s2} E_t\Delta p_{t+j+1}^2 + \gamma_x^s E_t x_{t+j} - \gamma_\pi^{s1} E_t i_{t+j}^*) \\ &= \frac{\alpha\gamma_\pi^{s1}}{\alpha\gamma_\pi^{s1} - (\alpha - \rho)} \Delta p_t^{1*} - \frac{\gamma_\pi^{s1}}{\alpha\gamma_\pi^{s1} - (\alpha - \rho)} \eta_t - \frac{\iota}{\gamma_\pi^{s1} - (1 - \rho)} - \frac{\gamma_\pi^{s1}}{\rho} \sum_{j=0}^{\infty} \left(\frac{1 - \gamma_\pi^{s1}}{\rho} \right)^j E_t\Delta p_{t+j+1}^{1*} \\ &\quad - \frac{\gamma_\pi^{s1}}{\rho} \sum_{j=0}^{\infty} \left(\frac{1 - \gamma_\pi^{s1}}{\rho} \right)^j \left(\frac{\gamma_\pi^{s2}}{\gamma_\pi^{s1}} E_t\Delta p_{t+j+1}^2 + \frac{\gamma_x^s}{\gamma_\pi^{s1}} E_t x_{t+j} - E_t i_{t+j}^* \right), \end{aligned}$$

where we use the fact $E_t\eta_{t+j} = 0$, $j = 1, 2, \dots$. Denoting $\hat{\iota} = -\frac{\iota}{\gamma_\pi^{s1} - (1 - \rho)} = -\frac{\iota}{(1 - \rho)(\delta\gamma_\pi - 1)}$ and $E_t f_{t+j} = -(E_t i_{t+j}^* - E_t\Delta p_{t+j+1}^{1*}) + \frac{\gamma_\pi^{s2}}{\gamma_\pi^{s1}} E_t\Delta p_{t+j+1}^2 + \frac{\gamma_x^s}{\gamma_\pi^{s1}} E_t x_{t+j} = -E_t r_{t+j}^* + \frac{1 - \delta}{\delta} E_t\Delta p_{t+j+1}^2 + \frac{\gamma_x}{\delta\gamma_\pi} E_t x_{t+j}$, (29) can be rewritten as follows.

$$z_{2,t} = \hat{\iota} + \frac{\alpha\gamma_\pi^{s1}}{\alpha\gamma_\pi^{s1} - (\alpha - \rho)} \Delta p_t^{1*} - \frac{\gamma_\pi^{s1}}{\alpha\gamma_\pi^{s1} - (\alpha - \rho)} \eta_t - \frac{\gamma_\pi^{s1}}{\rho} \sum_{j=0}^{\infty} \left(\frac{1 - \gamma_\pi^{s1}}{\rho} \right)^j E_t f_{t+j}, \quad (\text{a11})$$

Finally, plugging (a8) into (a3),

$$z_{3,t} = -i_{t-1}^* + v_t \quad (\text{a12})$$

Plug (a11) and (a12) into (a5) to get,

$$\begin{aligned} \Delta e_t = & \frac{\alpha\gamma_\pi^{s1}}{\alpha-\rho}\Delta p_t^1 - \frac{\alpha\gamma_\pi^{s1}}{\alpha-\rho}\Delta p_t^{1*} + \frac{\gamma_\pi^{s1}}{\alpha-\rho}\eta_t - \frac{\alpha\gamma_\pi^{s1} - (\alpha-\rho)}{\alpha-\rho}\bar{i} \\ & + \frac{\gamma_\pi^{s1}(\alpha\gamma_\pi^{s1} - (\alpha-\rho))}{\rho(\alpha-\rho)} \sum_{j=0}^{\infty} \left(\frac{1-\gamma_\pi^{s1}}{\rho}\right)^j E_t f_{t+j} + \frac{\alpha\gamma_\pi^{s1} - (\alpha-\rho)}{\alpha-\rho} i_{t-1}^* - \frac{\alpha\gamma_\pi^{s1} - (\alpha-\rho)}{\alpha-\rho} v_t \end{aligned} \quad (\text{a13})$$

Updating (29) once and applying the law of iterated expectations,

$$\begin{aligned} \Delta e_{t+1} = & \bar{i} + \frac{\alpha\gamma_\pi^{s1}}{\alpha-\rho}\Delta p_{t+1}^1 - \frac{\alpha\gamma_\pi^{s1}}{\alpha-\rho}\Delta p_{t+1}^{1*} + \frac{\alpha\gamma_\pi^{s1} - (\alpha-\rho)}{\alpha-\rho} i_t^* \\ & + \frac{\gamma_\pi^{s1}(\alpha\gamma_\pi^{s1} - (\alpha-\rho))}{\rho(\alpha-\rho)} \sum_{j=0}^{\infty} \left(\frac{1-\gamma_\pi^{s1}}{\rho}\right)^j E_t f_{t+j+1} + \omega_{t+1}, \end{aligned} \quad (\text{a14})$$

where,

$$\bar{i} = \frac{\alpha\gamma_\pi^{s1} - (\alpha-\rho)}{(\alpha-\rho)(\gamma_\pi^{s1} - (1-\rho))} \iota,$$

$$\begin{aligned} \omega_{t+1} = & \frac{\gamma_\pi^{s1}(\alpha\gamma_\pi^{s1} - (\alpha-\rho))}{\rho(\alpha-\rho)} \sum_{j=0}^{\infty} \left(\frac{1-\gamma_\pi^{s1}}{\rho}\right)^j (E_{t+1} f_{t+j+1} - E_t f_{t+j+1}) \\ & + \frac{\gamma_\pi^{s1}}{\alpha-\rho}\eta_{t+1} - \frac{\alpha\gamma_\pi^{s1} - (\alpha-\rho)}{\alpha-\rho} v_{t+1}, \end{aligned}$$

and,

$$E_t \omega_{t+1} = 0$$

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Table 1. GMM Estimation of the US Taylor Rule Estimation

Sample Period	$\hat{\gamma}_\pi$ (s.e.)	$\hat{\gamma}_x$ (s.e.)	$\hat{\rho}$ (s.e.)
1959:Q1-2003:Q4	1.7600 (0.4042)	0.2606 (0.0936)	0.8883 (0.0344)
1959:Q1-1979:Q2	0.7123 (0.0711)	0.6228 (0.1308)	0.6362 (0.0572)
1979:Q3-2003:Q4	2.9774 (0.3662)	0.2018 (0.2632)	0.8193 (0.0491)
1973:Q1-2003:Q4	2.6365 (1.5902)	0.6414 (0.5228)	0.9453 (0.0374)

Notes: i) Inflations are quarterly changes in log price level ($\ln p_t - \ln p_{t-1}$). ii) Quadratically detrended real GDP series are used for real GDP output deviations. iii) The set of instruments includes four lags of federal funds rate, inflation, output deviation, long-short interest rate spread, commodity price inflation, and M2 growth rate.

Table 2. CPI-Based Real Exchange Rates for the Post-Bretton Woods System

Country	$\hat{\alpha}$ (<i>s.e.</i>)	CI _{grid-t}	HL CI _{grid-t}	<i>J</i> -stat (<i>p.v.</i>)	#obs
Australia	0.950 (0.007)	[0.942,0.969]	[2.895,5.576]	10.27 (0.246)	124
Austria	0.942 (0.003)	[0.939,0.950]	[2.739,3.385]	12.89 (0.116)	104
Belgium	0.944 (0.002)	[0.942,0.949]	[2.895,3.337]	6.901 (0.547)	104
Canada	0.954 (0.009)	[0.943,0.980]	[2.947,8.577]	14.23 (0.076)	116
Denmark	0.939 (0.005)	[0.933,0.952]	[2.506,3.545]	6.597 (0.581)	124
Finland	0.958 (0.004)	[0.954,0.969]	[3.713,5.557]	4.811 (0.778)	84
France	0.946 (0.003)	[0.943,0.954]	[2.953,3.696]	7.338 (0.501)	104
Germany	0.943 (0.001)	[0.942,0.946]	[2.895,3.104]	9.015 (0.341)	104
Italy	0.954 (0.004)	[0.950,0.965]	[3.385,4.907]	9.269 (0.320)	104
Japan	0.928 (0.007)	[0.919,0.947]	[2.049,3.170]	5.800 (0.670)	124
Netherlands	0.939 (0.004)	[0.935,0.950]	[2.566,3.365]	5.294 (0.726)	104
New Zealand	0.947 (0.003)	[0.943,0.955]	[2.926,3.730]	8.127 (0.421)	124
Norway	0.941 (0.004)	[0.937,0.952]	[2.646,3.493]	8.569 (0.380)	124
Portugal	0.952 (0.005)	[0.947,0.966]	[3.176,5.025]	8.732 (0.365)	92
Spain	0.958 (0.008)	[0.950,0.982]	[3.385,9.280]	4.853 (0.773)	100
Sweden	0.968 (0.010)	[0.959,1.000]	[4.088, ∞]	7.165 (0.519)	116
Switzerland	0.937 (0.008)	[0.928,0.959]	[2.309,4.129]	6.110 (0.635)	113
UK	0.941 (0.005)	[0.935,0.955]	[2.578,3.721]	11.59 (0.171)	124
Median $\hat{\alpha}$: 0.945 [0.942,0.955]	Average $\hat{\alpha}$: 0.945 [0.941,0.961]		
Median HL	: 3.063 [2.895,3.725]	Average HL	: 3.165 [2.852,4.362]		

Notes: i) The US\$ is the base currency. ii) Quadratically detrended real GDP series are used for output deviations. iii) Sample periods are 1973.I ~ 1998.IV for Eurozone countries and are 1973.I ~ 2003.IV for non-Eurozone countries with some exceptions. iv) CI_{grid-t} denotes the 95% confidence intervals that were obtained by 2000 residual-based bootstrap replications on 200 grid points (Hansen, 1999). v) Median and Average half-lives were calculated from the corresponding $\hat{\alpha}$.

Table 3. PPI-Based Real Exchange Rates for the Post-Bretton Woods System

Country	$\hat{\alpha}$ (<i>s.e.</i>)	CI _{grid-t}	HL CI _{grid-t}	<i>J</i> -stat (<i>p.v.</i>)	#obs
Australia	0.958 (0.008)	[0.950,0.981]	[3.344,9.130]	12.77 (0.120)	124
Austria	0.961 (0.009)	[0.952,0.988]	[3.545,14.72]	9.262 (0.321)	104
Canada	0.995 (0.014)	[0.988,1.035]	[14.12, ∞]	9.278 (0.319)	116
Denmark	0.949 (0.004)	[0.945,0.960]	[3.046,4.234]	6.355 (0.607)	124
Finland	0.952 (0.006)	[0.946,0.969]	[3.146,5.576]	3.421 (0.905)	84
Germany	0.940 (0.003)	[0.937,0.948]	[2.654,3.251]	8.944 (0.347)	104
Japan	0.944 (0.004)	[0.940,0.955]	[2.781,3.738]	5.953 (0.653)	124
Netherlands	0.957 (0.006)	[0.951,0.974]	[3.456,6.656]	7.344 (0.500)	104
New Zealand	0.947 (0.004)	[0.941,0.957]	[2.850,3.952]	6.843 (0.554)	124
Norway	0.922 (0.009)	[0.910,0.946]	[1.844,3.134]	6.576 (0.583)	108
Spain	0.943 (0.008)	[0.935,0.965]	[2.562,4.907]	4.263 (0.833)	100
Sweden	0.948 (0.006)	[0.942,0.964]	[2.880,4.767]	8.254 (0.409)	116
Switzerland	0.949 (0.005)	[0.944,0.963]	[2.996,4.584]	4.738 (0.785)	113
UK	0.949 (0.007)	[0.941,0.968]	[2.855,5.397]	6.477 (0.594)	124
Median $\hat{\alpha}$: 0.949 [0.943,0.965]	Average $\hat{\alpha}$: 0.951 [0.944,0.970]		
Median HL	: 3.310 [2.937,4.836]	Average HL	: 3.449 [3.025,5.625]		

Notes: i) The US\$ is the base currency. ii) Quadratically detrended real GDP series are used for output deviations. iii) Sample periods are 1973.I \sim 1998.IV for Eurozone countries and are 1973.I \sim 2003.IV for non-Eurozone countries with some exceptions. iv) Belgium, France, and Italy are omitted because their post-Bretton Woods sample periods are same as their post-Volker era sample periods, and Portugal is omitted due to lack of enough observations. v) CI_{grid-t} denotes the 95% confidence intervals that were obtained by 2000 residual-based bootstrap replications on 200 grid points (Hansen, 1999). v) Median and Average half-lives were calculated from the corresponding $\hat{\alpha}$.

Table 4. Kim (2006)'s Data for the Post-Bretton Woods System

<i>Non-Service Goods Consumption Deflator</i>				
Country	$\hat{\alpha}$ (<i>s.e.</i>)	CI _{grid-t}	HL CI _{grid-t}	<i>J</i> -stat (<i>p.v.</i>)
Canada	0.999 (0.012)	[0.995,1.035]	[36.78, ∞]	6.780 (0.560)
France	0.941 (0.004)	[0.937,0.952]	[2.654,3.515]	8.391 (0.396)
Italy	0.949 (0.002)	[0.947,0.954]	[3.176,3.713]	6.902 (0.547)
Japan	0.937 (0.005)	[0.931,0.950]	[2.427,3.406]	9.938 (0.269)
Switzerland	0.931 (0.008)	[0.920,0.952]	[2.078,3.553]	15.19 (0.055)
UK	0.937 (0.007)	[0.929,0.956]	[2.346,3.869]	8.597 (0.377)
Median $\hat{\alpha}$: 0.939 [0.934,0.953]	Average $\hat{\alpha}$: 0.949 [0.943,0.967]	
Median HL	: 2.753 [2.536,3.631]	Average HL	: 3.310 [2.961,5.122]	

<i>Service Goods Consumption Deflator</i>				
Country	$\hat{\alpha}$ (<i>s.e.</i>)	CI _{grid-t}	HL CI _{grid-t}	<i>J</i> -stat (<i>p.v.</i>)
Canada	0.956 (0.006)	[0.948,0.974]	[3.264,6.656]	16.62 (0.034)
France	0.945 (0.002)	[0.943,0.950]	[2.947,3.406]	10.89 (0.207)
Italy	0.948 (0.003)	[0.945,0.956]	[3.057,3.869]	4.568 (0.803)
Japan	0.942 (0.004)	[0.938,0.953]	[2.685,3.584]	9.493 (0.302)
Switzerland	0.968 (0.006)	[0.961,0.986]	[4.311,11.86]	6.053 (0.641)
UK	0.940 (0.004)	[0.935,0.951]	[2.587,3.428]	8.438 (0.392)
Median $\hat{\alpha}$: 0.947 [0.944,0.955]	Average $\hat{\alpha}$: 0.950 [0.945,0.962]	
Median HL	: 3.152 [3.001,3.721]	Average HL	: 3.367 [3.057,4.431]	

Notes: i) The US\$ is the base currency. ii) Quadratically detrended real GDP series are used for output deviations. iii) CI_{grid-t} denotes the 95% confidence intervals that were obtained by 2000 residual-based bootstrap replications on 200 grid points (Hansen, 1999). iv) Median and Average half-lives were calculated from the corresponding $\hat{\alpha}$.

Table 5. CPI-Based Real Exchange Rates for the Post-Volker Era

Country	$\hat{\alpha}$ (<i>s.e.</i>)	CI _{grid-t}	HL CI _{grid-t}	<i>J</i> -stat (<i>p.v.</i>)	#obs
Australia	0.875 (0.025)	[0.838,0.941]	[0.983,2.845]	7.194 (0.516)	98
Austria	0.807 (0.007)	[0.797,0.824]	[0.762,0.897]	8.465 (0.389)	78
Belgium	0.810 (0.005)	[0.802,0.822]	[0.787,0.884]	7.842 (0.449)	78
Canada	0.962 (0.023)	[0.937,1.033]	[2.654, ∞]	7.898 (0.444)	98
Denmark	0.814 (0.005)	[0.806,0.826]	[0.804,0.905]	10.61 (0.225)	98
Finland	0.886 (0.021)	[0.857,0.943]	[1.120,2.963]	5.551 (0.697)	78
France	0.818 (0.004)	[0.812,0.828]	[0.832,0.916]	8.511 (0.385)	78
Germany	0.817 (0.004)	[0.811,0.827]	[0.827,0.911]	6.872 (0.550)	78
Italy	0.833 (0.007)	[0.823,0.851]	[0.891,1.070]	6.913 (0.546)	78
Japan	0.809 (0.011)	[0.792,0.836]	[0.744,0.968]	12.52 (0.130)	98
Netherlands	0.827 (0.005)	[0.820,0.839]	[0.871,0.990]	7.972 (0.436)	78
New Zealand	0.829 (0.012)	[0.809,0.857]	[0.816,1.124]	6.401 (0.602)	98
Norway	0.825 (0.006)	[0.816,0.840]	[0.852,0.991]	9.080 (0.336)	98
Portugal	0.827 (0.008)	[0.815,0.847]	[0.849,1.041]	6.060 (0.640)	78
Spain	0.909 (0.032)	[0.863,0.997]	[1.175,55.81]	5.058 (0.751)	78
Sweden	0.948 (0.028)	[0.910,1.028]	[1.844, ∞]	6.672 (0.572)	90
Switzerland	0.824 (0.006)	[0.815,0.838]	[0.846,0.982]	12.71 (0.122)	98
UK	0.808 (0.009)	[0.795,0.830]	[0.753,0.928]	13.62 (0.092)	98
Median $\hat{\alpha}$: 0.826 [0.815,0.839]	Average $\hat{\alpha}$: 0.846 [0.829,0.878]		
Median HL	: 0.906 [0.847,0.990]	Average HL	: 1.036 [0.923,1.333]		

Notes: i) The US\$ is the base currency. ii) Quadratically detrended real GDP series are used for output deviations. iii) Sample periods are 1979.III \sim 1998.IV for Eurozone countries and are 1979.III \sim 2003.IV for non-Eurozone countries with some exceptions. iv) CI_{grid-t} denotes the 95% confidence intervals that were obtained by 2000 residual-based bootstrap replications on 200 grid points (Hansen, 1999). v) Median and Average half-lives were calculated from the corresponding $\hat{\alpha}$.

Table 6. PPI-Based Real Exchange Rates for the Post-Volker Era

Country	$\hat{\alpha}$ (<i>s.e.</i>)	CI _{grid-t}	HL CI _{grid-t}	<i>J</i> -stat (<i>p.v.</i>)	#obs
Australia	0.895 (0.023)	[0.863,0.959]	[1.171,4.109]	5.257 (0.730)	98
Austria	0.859 (0.022)	[0.828,0.917]	[0.916,1.990]	6.530 (0.588)	78
Belgium	0.830 (0.014)	[0.809,0.866]	[0.819,1.202]	9.385 (0.311)	76
Canada	0.894 (0.030)	[0.852,0.983]	[1.082,9.988]	6.470 (0.595)	98
Denmark	0.853 (0.012)	[0.835,0.883]	[0.964,1.391]	7.772 (0.456)	98
Finland	0.906 (0.021)	[0.880,0.966]	[1.354,4.980]	2.450 (0.964)	78
France	0.846 (0.015)	[0.825,0.884]	[0.900,1.408]	4.010 (0.856)	76
Germany	0.868 (0.009)	[0.856,0.891]	[1.114,1.501]	6.056 (0.641)	78
Italy	0.829 (0.016)	[0.807,0.871]	[0.806,1.249]	7.207 (0.514)	72
Japan	0.820 (0.006)	[0.811,0.835]	[0.828,0.958]	11.51 (0.174)	98
Netherlands	0.893 (0.015)	[0.873,0.933]	[1.275,2.503]	5.992 (0.648)	78
New Zealand	0.887 (0.019)	[0.856,0.936]	[1.112,2.607]	8.626 (0.375)	98
Norway	0.800 (0.009)	[0.786,0.822]	[0.721,0.882]	8.119 (0.422)	98
Spain	0.889 (0.022)	[0.860,0.951]	[1.151,3.442]	4.747 (0.784)	78
Sweden	0.876 (0.022)	[0.845,0.934]	[1.030,2.546]	6.467 (0.595)	90
Switzerland	0.839 (0.014)	[0.819,0.874]	[0.866,1.287]	7.902 (0.443)	98
UK	0.905 (0.031)	[0.863,1.003]	[1.174, ∞]	5.036 (0.754)	98
Median $\hat{\alpha}$: 0.868 [0.845,0.917]	Average $\hat{\alpha}$: 0.864 [0.839,0.912]		
Median HL	: 1.224 [1.030,1.990]	Average HL	: 1.186 [0.989,1.882]		

Notes: i) The US\$ is the base currency. ii) Quadratically detrended real GDP series are used for output deviations. iii) Sample periods are 1979.III \sim 1998.IV for Eurozone countries and are 1979.III \sim 2003.IV for non-Eurozone countries with some exceptions. iv) CI_{grid-t} denotes the 95% confidence intervals that were obtained by 2000 residual-based bootstrap replications on 200 grid points (Hansen, 1999). v) Median and Average half-lives were calculated from the corresponding $\hat{\alpha}$.

Table 7. Kim (2006)'s Data for the Post-Volker Era

<i>Non-Service Goods Consumption Deflator</i>				
Country	$\hat{\alpha}$ (<i>s.e.</i>)	CI _{grid-t}	HL CI _{grid-t}	<i>J</i> -stat (<i>p.v.</i>)
Canada	0.921 (0.028)	[0.886,1.012]	[1.430, ∞]	9.845 (0.276)
France	0.809 (0.009)	[0.796,0.831]	[0.759,0.938]	6.749 (0.564)
Italy	0.812 (0.005)	[0.805,0.824]	[0.798,0.897]	6.750 (0.564)
Japan	0.834 (0.007)	[0.824,0.851]	[0.896,1.077]	7.954 (0.438)
Switzerland	0.937 (0.021)	[0.913,1.000]	[1.899, ∞]	4.298 (0.829)
UK	0.819 (0.009)	[0.806,0.841]	[0.803,1.002]	10.55 (0.229)
Median $\hat{\alpha}$: 0.827 [0.815,0.846]	Average $\hat{\alpha}$: 0.855 [0.838,0.893]	
Median HL	: 0.909 [0.847,1.038]	Average HL	: 1.109 [0.982,1.537]	

<i>Service Goods Consumption Deflator</i>				
Country	$\hat{\alpha}$ (<i>s.e.</i>)	CI _{grid-t}	HL CI _{grid-t}	<i>J</i> -stat (<i>p.v.</i>)
Canada	0.912 (0.015)	[0.891,0.956]	[1.503,3.842]	13.49 (0.096)
France	0.812 (0.009)	[0.809,0.844]	[0.817,1.025]	8.787 (0.361)
Italy	0.783 (0.023)	[0.749,0.840]	[0.600,0.997]	6.082 (0.683)
Japan	0.807 (0.005)	[0.800,0.819]	[0.776,0.870]	5.621 (0.690)
Switzerland	0.953 (0.025)	[0.927,1.023]	[2.289, ∞]	4.351 (0.824)
UK	0.846 (0.013)	[0.828,0.879]	[0.918,1.342]	6.871 (0.551)
Median $\hat{\alpha}$: 0.834 [0.818,0.862]	Average $\hat{\alpha}$: 0.854 [0.834,0.894]	
Median HL	: 0.955 [0.865,1.164]	Average HL	: 1.097 [0.955,1.542]	

Notes: i) The US\$ is the base currency. ii) Quadratically detrended real GDP series are used for output deviations. iii) CI_{grid-t} denotes the 95% confidence intervals that were obtained by 2000 residual-based bootstrap replications on 200 grid points (Hansen, 1999). iv) Median and Average half-lives were calculated from the corresponding $\hat{\alpha}$.

Table 8. Univariate Estimation Results for the Post-Bretton Woods System

Country	CPI-Based Rates		PPI-Based Rates	
	$\hat{\alpha}$ (<i>s.e.</i>)	CI _{grid-t}	$\hat{\alpha}$ (<i>s.e.</i>)	CI _{grid-t}
Australia	0.956 (0.025)	[0.923,1.030]	0.909 (0.036)	[0.856,1.023]
Austria	0.929 (0.035)	[0.881,1.035]	0.913 (0.040)	[0.858,1.033]
Belgium	0.944 (0.032)	[0.903,1.040]		
Canada	0.970 (0.019)	[0.946,1.028]	0.909 (0.037)	[0.854,1.024]
Denmark	0.936 (0.032)	[0.892,1.031]	0.937 (0.033)	[0.891,1.033]
Finland	0.948 (0.036)	[0.906,1.051]	0.929 (0.041)	[0.878,1.051]
France	0.929 (0.036)	[0.879,1.038]		
Germany	0.925 (0.037)	[0.874,1.037]	0.927 (0.037)	[0.878,1.038]
Italy	0.928 (0.037)	[0.879,1.038]		
Japan	0.950 (0.025)	[0.914,1.027]	0.941 (0.030)	[0.901,1.030]
Netherlands	0.924 (0.037)	[0.872,1.036]	0.936 (0.035)	[0.890,1.040]
New Zealand	0.978 (0.009)	[0.965,1.006]	0.981 (0.008)	[0.969,1.008]
Norway	0.926 (0.034)	[0.878,1.029]	0.894 (0.041)	[0.832,1.025]
Portugal	0.961 (0.029)	[0.929,1.047]		
Spain	0.953 (0.029)	[0.918,1.041]	0.939 (0.034)	[0.894,1.041]
Sweden	0.984 (0.026)	[0.960,1.050]	0.941 (0.035)	[0.895,1.040]
Switzerland	0.920 (0.037)	[0.866,1.032]	0.920 (0.037)	[0.867,1.032]
UK	0.918 (0.037)	[0.864,1.029]	0.951 (0.030)	[0.913,1.037]
Median $\hat{\alpha}$: 0.940 [0.897,1.036]		0.933 [0.884,1.033]	
Median HL	: 2.801 [1.594, ∞]		2.500 [1.405, ∞]	

Notes: i) The US\$ is the base currency. ii) Quadratically detrended real GDP series are used for output deviations. iii) Sample periods are 1973.I ~ 1998.IV for Eurozone countries and are 1973.I ~ 2003.IV for non-Eurozone countries with some exceptions. iv) For PPI-based real exchange rates, Belgium, France, and Italy are omitted because their post-Bretton Woods sample periods are same as their post-Volker era sample periods, and Portugal is omitted due to lack of enough observations. v) CI_{grid-t} denotes the 95% confidence intervals that were obtained by 2000 residual-based bootstrap replications on 200 grid points (Hansen, 1999). vi) Median and Average half-lives were calculated from the corresponding $\hat{\alpha}$.

Table 9. Univariate Estimation Results for the Post-Volker Era

Country	CPI-Based Rates		PPI-Based Rates	
	$\hat{\alpha}$ (<i>s.e.</i>)	CI _{grid-t}	$\hat{\alpha}$ (<i>s.e.</i>)	CI _{grid-t}
Australia	0.935 (0.033)	[0.889,1.036]	0.913 (0.041)	[0.856,1.038]
Austria	0.936 (0.038)	[0.885,1.051]	0.919 (0.043)	[0.860,1.048]
Belgium	0.919 (0.038)	[0.865,1.038]	0.904 (0.047)	[0.838,1.047]
Canada	0.971 (0.023)	[0.948,1.039]	0.914 (0.041)	[0.857,1.039]
Denmark	0.929 (0.035)	[0.880,1.036]	0.943 (0.035)	[0.898,1.045]
Finland	0.945 (0.037)	[0.901,1.054]	0.929 (0.041)	[0.876,1.054]
France	0.918 (0.041)	[0.860,1.043]	0.924 (0.043)	[0.868,1.053]
Germany	0.910 (0.042)	[0.851,1.040]	0.936 (0.039)	[0.888,1.052]
Italy	0.936 (0.039)	[0.886,1.053]	0.944 (0.039)	[0.898,1.059]
Japan	0.947 (0.032)	[0.908,1.042]	0.938 (0.034)	[0.894,1.040]
Netherlands	0.902 (0.043)	[0.840,1.038]	0.942 (0.038)	[0.895,1.053]
New Zealand	0.946 (0.017)	[0.921,0.999]	0.948 (0.015)	[0.924,0.996]
Norway	0.922 (0.037)	[0.870,1.036]	0.884 (0.045)	[0.816,1.027]
Portugal	0.969 (0.029)	[0.945,1.055]		
Spain	0.954 (0.031)	[0.917,1.047]	0.942 (0.035)	[0.897,1.049]
Sweden	0.970 (0.030)	[0.935,1.055]	0.941 (0.037)	[0.894,1.049]
Switzerland	0.916 (0.039)	[0.859,1.035]	0.920 (0.039)	[0.866,1.037]
UK	0.908 (0.043)	[0.847,1.038]	0.937 (0.037)	[0.889,1.045]
Median $\hat{\alpha}$: 0.935 [0.886,1.039]		0.936 [0.888,1.047]	
Median HL	: 2.578 [1.432, ∞]		2.620 [1.459, ∞]	

Notes: i) The US\$ is the base currency. ii) Quadratically detrended real GDP series are used for output deviations. iii) Sample periods are 1979.III \sim 1998.IV for Eurozone countries and are 1979.III \sim 2003.IV for non-Eurozone countries with some exceptions. iv) For PPI-based real exchange rates, Portugal is omitted due to lack of enough observations. v) CI_{grid-t} denotes the 95% confidence intervals that were obtained by 2000 residual-based bootstrap replications on 200 grid points (Hansen, 1999). vi) Median and Average half-lives were calculated from the corresponding $\hat{\alpha}$.