A Note on Real Exchange Rate Dynamics and the Taylor Rule

Hyeongwoo Kim*

Auburn University

February 2009

(Preliminary Draft)

Abstract

In their recent article, Kehoe and Midrigan (2007) derive an AR(1) representation for the real exchange rate. Their model implies that the persistence parameter of the real exchange rate equals a measure of price-stickiness in Calvo-pricing models under fairly general assumptions. This note demonstrates that such a representation can be consistent with a model where money supply is endogenously determined by the Taylor Rule along the line of Woodford (2007).

Keywords: Cash-in-Advance, Local-Currency Pricing, Pricing-to-Market, Taylor Rule

JEL Classification: E40, F31, F41

1 Introduction

Kehoe and Midrigan (2007) recently show that real exchange rates can be represented by a closed-form first order autoregressive process when consumers have log utility for consumption, leisure is separable from consumption, and nominal exchange rates obey a random walk process. This AR(1) representation is interesting because the persistence parameter coincides with a measure of price-stickiness, the probability that firms keep their prices unchanged in any given period in Calvo-pricing models. Relaxing these assumptions yield no closed-form representations but Kehoe and Midrigan report strong positive correlations between the persistence parameter and the Calvo-probability parameter.

*Department of Economics, Auburn University, 415 W. Magnolia Ave., Auburn, AL 36849. Tel: (334) 844-2928. Fax: (334) 844-4615. Email: gmmkim@gmail.com

This note demonstrates that the results of Kehoe and Midrigan can be consistent with a model where money supply is endogenously determined as the central bank changes interest rate by the Taylor Rule along the line of Woodford (2007). Following Kehoe and Midrigan, we employ a Cash-in-Advance assumption to derive the AR(1) process for real exchange rates but separate consumptions from real GDP by introducing government consumption sector, which is enough to make Kehoe and Midrigan's results to be consistent with models with the Taylor Rule.

The present note is organized as follows. Section 2 describes the model and Section 3 derives an AR(1) representation of real exchange rates. Section 4 concludes.

2 The Model

2.1 Preferences

There are two symmetrical countries, a home and a foreign country. The representative consumer solves,

$$Max\ \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(C_t, N_t) \right\}, \tag{1}$$

where $\mathbb{E}_0$ denotes the conditional expectation operator at time $t = 0$, C_t is the aggregate consumption index, N_t denotes hours of work, and β is discount factor. The flow budget constraint expressed in units of the home currency is,

$$M_{c,t} + Q_{t,t+1}B_{t+1} \leq B_t + R_{t-1}W_{t-1}N_{t-1} + M_{c,t-1} - P_{t-1}C_{t-1} + T_t, \tag{2}$$

$$M_{c,t} \geq P_t C_t, \tag{3}$$

where $M_{c,t}$ is the nominal money holding of the consumer and B_{t+1} denotes the risk-free nominal bond holding that pays one unit of home currency in the beginning of period $t+1$ and $Q_{t,t+1}$ is the observed price of it at time t. Following Kehoe and Midrigan (2007), we assume that the government pays interest (R_{t-1}) on nominal wages $W_{t-1}N_{t-1}$ to prevent distortion in the consumption-labor optimality condition. Unused money holding from last period $(M_{t-1} - P_{t-1}C_{t-1})$ is carried over to period t. T_t is a lump-sum income that may contains government transfers and nominal dividends from ownership of firms. Finally, the equation (3) is the cash-in-advance (CIA) constraint. We also impose the following transversality condition.

$$\lim_{T\to\infty} \mathbb{E}_t B_T \geq 0,\ \ t = 0, 1, 2, \cdots \tag{4}$$

Aggregate consumption C_t is a composite of consumption of a unit mass of goods, indexed $i \in [0, 1]$,

$$C_t = \left[\int_0^1 C_{i,t}^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}, \tag{5}$$

where goods indexed $i \in [0, \frac{1}{2})$ are produced in the home country while the ones indexed $i \in [\frac{1}{2}, 1]$ are produced in the foreign country. The corresponding aggregate price index is,

$$P_t = \left[\int_0^1 P_{i,t}^{1-\theta} di\right]^{\frac{1}{1-\theta}} \tag{6}$$

The foreign representative consumer solves a similar optimization problem.[1] Foreign consumptions and prices are analogously defined and denoted by a superscript asterisk ($*$).

Assume a utility function of the following form.

$$u(C_t, N_t) = \ln C_t - \psi N_t, \tag{7}$$

where ψ is disutility parameter of labor. Let E_t denote the nominal exchange rate as the unit price of foreign currency in terms of domestic currency. The first order conditions are,

$$\frac{W_t}{P_t} = \psi C_t,\ \frac{W_t^*}{P_t^*} = \psi C_t^* \tag{8}$$

$$Q_{t,t+1} = \beta \mathbb{E}_t \left(\frac{P_t C_t}{P_{t+1} C_{t+1}}\right) = \beta \mathbb{E}_t \left(\frac{E_t P_t^* C_t^*}{E_{t+1} P_{t+1}^* C_{t+1}^*}\right) \tag{9}$$

$$M_{c,t} = P_t C_t,\ M_{c,t}^* = P_t^* C_t^* \tag{10}$$

The real exchange rate is,

$$S_t = \frac{E_t P_t^*}{P_t} \tag{11}$$

[1] Recall that the nominal bonds are denominated in the home currency. Therefore, the flow budget constraint of foreign consumers is, $M_{c,t}^* + Q_{t,t+1} B_{t+1}^*/E_t \leq B_t^*/E_t + E_{t-1} R_{t-1} W_{t-1}^* N_{t-1}^*/E_t + M_{c,t-1}^* - P_{t-1}^* C_{t-1}^* + T_t^*$, where E_t is the nominal exchange rate.

From (9),

$$S_t = \frac{C_t}{C_t^*}, \tag{12}$$

where we normalize $S_0 C_0 / C_0^* = 1$ for simplicity. Combining (12) with (10), we get,

$$E_t = \frac{M_{c,t}}{M_{c,t}^*} \tag{13}$$

2.2 Firms

Firm i produces output $Y_{i,t}$ using a linear technology,

$$Y_{i,t} = N_{i,t}, \tag{14}$$

where $N_{i,t}$ is the labor input by the firm.

Firms are monopolistic competitors and set their prices in the local currency (Local-Currency Pricing). Each home firm $i \in [0, \frac{1}{2})$ sets prices $P_{i,t}$ and $P_{i,t}^*$ separately for the goods in the home and the foreign country, respectively (Pricing-to-Market). Following Calvo (1993), each firm is assumed to reset its price with probability $1 - \alpha$ in each period, independent of the time since its last price adjustment. That is, firms leave their prices unchanged for $\frac{1}{1-\alpha}$ periods on average.

The problem of a price-adjusting home firm in the home market at time t is,

$$\underset{P_{H,i,t}}{Max} \ \mathbb{E}_t \sum_{j=0}^{\infty} \alpha^j Q_{t,t+j} \left(P_{H,i,t} - W_{t+j}\right) \left(C_{H,i,t+j} + G_{H,i,t+j}\right), \tag{15}$$

where $G_{H,i,T}$ is government spending on domestic consumption good $i \in [0, \frac{1}{2})$. We assume the demand function of $G_{H,i,T}$ is defined as (5). The optimization problem (15) is, then, subject to

$$\left(C_{H,i,t+j} + G_{H,i,t+j}\right) = \left(\frac{P_{H,i,t}}{P_{i,t+j}}\right)^{-\theta} \left(C_{i,t+j} + G_{H,t+j}\right) = \left(\frac{P_{H,i,t}}{P_{t+j}}\right)^{-\theta} \left(C_{t+j} + G_{t+j}\right) \tag{16}$$

The necessary condition is,

$$\mathbb{E}_t \sum_{j=0}^{\infty} \alpha^j Q_{t,t+j} \left(\frac{P_{H,i,t}}{P_{t+j}}\right)^{-\theta} (C_{t+j} + G_{t+j}) \tag{17}$$
$$= \frac{\theta}{\theta - 1} \mathbb{E}_t \sum_{j=0}^{\infty} \alpha^j Q_{t,t+j} \left(\frac{W_{t+j}}{P_{H,i,t}}\right) \left(\frac{P_{H,i,t}}{P_{t+j}}\right)^{-\theta} (C_{t+j} + G_{t+j})$$

Similarly, the necessary condition of a foreign firm in the home market is,

$$\mathbb{E}_t \sum_{j=0}^{\infty} \alpha^j Q_{t,t+j} \left(\frac{P_{F,i,t}}{P_{t+j}}\right)^{-\theta} (C_{t+j} + G_{t+j}) \tag{18}$$
$$= \frac{\theta}{\theta - 1} \mathbb{E}_t \sum_{j=0}^{\infty} \alpha^j Q_{t,t+j} \left(\frac{E_{t+j} W_{t+j}}{P_{F,i,t}}\right) \left(\frac{P_{F,i,t}}{P_{t+j}}\right)^{-\theta} (C_{t+j} + G_{t+j})$$

2.3 Governments

We assume that government purchases in the home country are determined *exogenously* and are under the CIA restriction.

$$M_{g,t} \geq P_t G_t = \int_0^1 P_{i,t} G_{i,t} di \tag{19}$$

Total transfers to consumers are,

$$T_t = M_t - M_{t-1} - (R_{t-1} - 1) W_{t-1} N_{t-1}, \tag{20}$$

where

$$M_t = M_{c,t} + M_{g,t} \tag{21}$$

Total money supply M_t is endogenously determined with,

$$\ln(M_t / P_t) = L(i_t, Y_t), \tag{22}$$

where i_t is the nominal net interest rate set by the Taylor Rule and real GDP is,

$$Y_t = C_t + G_t \tag{23}$$

Note that the Taylor Rule is about i_t and Y_t. Being agnostic about the stochastic process of government sector, this allows $\ln(M - M_{g,t})$ to be a random walk process. That is, the log money supply for consumptions goods obeys a random walk process,

$$\mu_{c,t} = \varepsilon_{c,t} \tag{24}$$

where $\mu_{c,t} = \ln(M_{c,t}/M_{c,t-1})$.

The stochastic process of foreign log money supply for the foreign consumers is similarly defined,

$$\mu^*_{c,t} = \varepsilon^*_{c,t} \tag{25}$$

3 Real Exchange Rate Dynamics

Note that $p_{H,i,t} = p_{H,t}, \forall i \in [0, 1/2]$ due to symmetry. For the same reason, $p_{F,i,t} = p_{F,t}, \forall i \in (1/2, 1]$. Let $\hat{x}_t$ denotes log-deviation of a generic variable X_t. Log-linearizing (17),

$$\hat{p}_{H,t} = (1-\alpha\beta)\,\mathbb{E}_t \sum_{j=0}^{\infty} (\alpha\beta)^j\,\hat{w}_{t+j} = (1-\alpha\beta)\,\mathbb{E}_t \sum_{j=0}^{\infty} (\alpha\beta)^j\,\hat{m}_{c,t+j}, \tag{26}$$

where the last equality comes from (8) and (10). (26), then, can be recursively written,

$$\hat{p}_{H,t} = \alpha\beta\mathbb{E}_t\hat{p}_{H,t+1} + (1-\alpha\beta)\hat{m}_{c,t} \tag{27}$$

Analogously,

$$\hat{p}_{F,t} = (1-\alpha\beta)\,\mathbb{E}_t \sum_{j=0}^{\infty} (\alpha\beta)^j \left(\hat{e}_{t+j} + \hat{w}^*_{t+j}\right) = (1-\alpha\beta)\,\mathbb{E}_t \sum_{j=0}^{\infty} (\alpha\beta)^j\,\hat{m}_{c,t+j}, \tag{28}$$

by (8), (10), and (13). Recursively,

$$\hat{p}_{F,t} = \alpha\beta\mathbb{E}_t\hat{p}_{F,t+1} + (1-\alpha\beta)\hat{m}_{c,t}, \tag{29}$$

Using (6), p_t can be approximated as,

$$\hat{p}_t = \alpha \hat{p}_{t-1} + (1-\alpha)\left[\delta \hat{p}_{H,t} + (1-\delta)\hat{p}_{F,t}\right], \tag{30}$$

where $\delta = \frac{P_{H,t}^{1-\theta}}{P_{H,t}^{1-\theta} + P_{F,t}^{1-\theta}}$ is a weight parameter on home products. The foreign counterpart is,

$$\hat{p}_t^* = \alpha \hat{p}_{t-1}^* + (1-\alpha)\left[\delta \hat{p}_{H,t}^* + (1-\delta)\hat{p}_{F,t}^*\right] \tag{31}$$

Note that $\hat{p}_{H,t} = \hat{p}_{F,t} = \hat{q}_t$ and $\hat{p}_{H,t}^* = \hat{p}_{F,t}^* = \hat{q}_t^*$ under the current framework, where $\hat{q}_t$ and $\hat{q}_t^*$ are newly-reset log price deviations and $\delta = \frac{1}{2}$. Then, (30) and (31) can be rewritten as,

$$\hat{p}_t = \alpha \hat{p}_{t-1} + (1-\alpha)\hat{q}_t \tag{32}$$

$$\hat{p}_t^* = \alpha \hat{p}_{t-1}^* + (1-\alpha)\hat{q}_t^* \tag{33}$$

Subtracting $\hat{m}_{c,t}$ from each side of (26),

$$\hat{q}_t - \hat{m}_{c,t} = (1-\alpha\beta)\,\mathbb{E}_t \sum_{j=0}^{\infty} (\alpha\beta)^j \left(\hat{m}_{c,t+j} - \hat{m}_{c,t}\right) = 0, \tag{34}$$

which implies,

$$\hat{q}_t = \hat{m}_{c,t} \tag{35}$$

Subtracting $\hat{m}_{c,t}$ from each side of (32),

$$\begin{aligned} \hat{p}_t - \hat{m}_{c,t} &= \alpha(\hat{p}_{t-1} - \hat{m}_{c,t-1}) - \alpha(\hat{m}_{c,t} - \hat{m}_{c,t-1}) + (1-\alpha)(\hat{q}_t - \hat{m}_{c,t}) \\ &= \alpha(\hat{p}_{t-1} - \hat{m}_{c,t-1}) - \alpha\varepsilon_{c,t}, \end{aligned} \tag{36}$$

where the last equality uses (24) and (35). Similarly,

$$\hat{p}_t^* - \hat{m}_{c,t}^* = \alpha(\hat{p}_{t-1}^* - \hat{m}_{c,t-1}^*) - \alpha\varepsilon_{c,t}^* \tag{37}$$

From (13),

$$\hat{e}_t = \hat{m}_{c,t} - \hat{m}_{c,t}^*, \tag{38}$$

From (11), (36), (37), and (38),

$$\begin{aligned}\hat{s}_t &= \hat{e}_t + \hat{p}_t^* - \hat{p}_t \\ &= \alpha(\hat{p}_{t-1}^* - \hat{p}_{t-1} - \hat{m}_{c,t-1}^* + \hat{m}_{c,t-1}) - \alpha(\varepsilon_{c,t}^* - \varepsilon_{c,t}) \\ &= \alpha(\hat{p}_{t-1}^* - \hat{p}_{t-1} + \hat{e}_{t-1}) - \alpha(\varepsilon_{c,t}^* - \varepsilon_{c,t}),\end{aligned}$$

therefore,

$$\hat{s}_t = \alpha \hat{s}_{t-1} + \alpha \eta_t, \tag{39}$$

which implies,

$$s_{t+1} = d + \alpha s_t + \varepsilon_{t+1}, \tag{40}$$

where $\frac{d}{1-\alpha}$ is long-run equilibrium PPP and $\varepsilon_{t+1} = \alpha\eta_{t+1} = \alpha(\varepsilon_{c,t+1} - \varepsilon_{c,t+1}^*)$.

4 Conclusion

This note describes an economy where Kehoe and Midrigan's (2007) model can be consistent with a model with the Taylor Rule along the line of Woodford (2007). For this purpose, we introduce a government consumption sector that separates consumption from real GDP. With this modification, we derive an AR(1) representation of real exchange rates where the persistence parameter equals the probability of firms that do not update their prices in any given period as Kehoe and Midrigan's model implies. This result is quite useful because it is convenient to have an AR(1) representation for empirical investigation of real exchange rates dynamics based on New Keynesian general equilibrium models.

References

Kehoe, P. J., and V. Midrigan (2007): "Stikcy Prices and Sectoral Real Exchange Rates.," Federal Reserve Bank of Minneapolis Working Paper 656.

Woodford, M. (2007): "How Important Is Money in the Conduct of Monetary Policy," *Journal of Money, Credit, and Banking*, forthcoming.