Examining the Evidence for Purchasing Power Parity Under the Current Float by Recursive Mean Adjustment

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Abstract

This paper revisits the empirical evidence on real exchange rate dynamics by employing recently developed recursive mean adjustment (RMA) method (So and Shin, 1999). Our major findings are twofold. First, using the current float quarterly real exchange rates relative to the US dollar we find that the unit-root test with RMA rejects the null of unit root for 16 out of 20 industrialized countries. Second, we find that the RMA estimator and computationally simple asymptotic confidence intervals can provide useful information regarding the real exchange rate dynamics. Our findings casts doubt on Murray and Papell's (2002) claim that univariate approaches provide virtually no useful information on the size of real exchange rate half-lives.

Keywords: Purchasing Power Parity, Recursive Mean Adjustment, Half-Life

JEL Classification: C22, F31

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1 Introduction

Purchasing power parity (PPP) asserts that price levels across countries should be equalized over time once converted into the same currency. PPP serves as a key building block for many open economy macro models. Despite its popularity and extensive research, empirical validity of PPP remains inconclusive due to mixed empirical evidence.

Testing for long-run PPP is typically carried over by implementing unit root tests for real exchange rates. Studies employing conventional augmented Dickey-Fuller (ADF) tests find very little evidence of PPP with the post Bretton Woods system (current float) real exchange rates. It is well known that the ADF tests has very low power when time span of the data is relatively short. Indeed, empirical studies that use long-horizon data, rather than using the current float data, find more favorable evidence for PPP, among others, Taylor (2002a) and Rogoff (1996). In an effort to overcome the power problem, an array of researches employed panel unit-root tests for the current float data and report evidence in favor of PPP. It should be noted, however, that panel unit-root tests may be oversized (Phillips and Sul, 2003).[1] Therefore, it is not clear that panel approaches with the current float data solve the power problem.

Another important issue we note is the following. It is a well-known statistical fact that the least squares (LS) estimator for autoregressive (AR) processes suffers from serious small-sample bias when the stochastic process includes a non-zero intercept and/or deterministic time trend. The bias can be substantial especially when the process is highly persistent (Andrews, 1993).

Since the pioneering work of Kendall (1954), many bias-correction methods have been developed. Andrews (1993) proposed a method to obtain the exactly median-unbiased estimator for AR(1) process with normal errors. Andrews and Chen (1994) extended the work of Andrews (1993) and develop approximately median-unbiased estimator for AR(p) processes. Hansen (1999) developed a nonparametric bias correction method of grid bootstrap (GT) that is robust to distributional assumptions.

Murray and Papell (2002) employ methods proposed by Andrews(1993) and Andresw and Chen (1994) to correct for the downward bias in the persistent parameter estimates and find that confi-

[1]Phillips and Sul (2003) show that conventional panel unit-root tests tend to reject the null of unit root too often in presence of cross-section dependence. O'Connell (1998) finds much weaker evidence for PPP controlling for cross-section dependence.

dence intervals for the half-lives of most current float real exchange rates extend to positive infinity. Based on this, they conclude that the univariate estimation methods provides no useful information on the real exchange rates dynamics.

We revisits these issues by employing an alternative estimator, recursive mean adjustment (RMA) by So and Shin (1999), that belongs to a class of (approximately) mean-unbiased estimators. The RMA estimator is computationally convenient to implement yet powerful and has been employed in various studies. For instance, Choi *et al.* (2008) and Sul *et al.* (2005) apply RMA to mitigate prewhitening bias in heteroskedasticity and autocorrelation consistent estimation. Taylor (2002b) employs RMA for seasonal unit root test and found superior size and power properties. Cook (2002) applied RMA to correct a severe oversize problem of the Dicky-Fuller test in the presence of level break.

Our major findings are twofold. First, we test the null of unit root for 20 current float quarterly real exchange rates relative to the US dollar using a more powerful RMA-based unit root test (Shin and So, 2001) and find that the test rejects the null for 16 countries at the 10% significance level while the conventional ADF test rejects the null only for 5. Second, unlike Murray and Papell (2002), we obtain compact confidence intervals for the half-lives of those countries that pass the RMA-based unit root test.

The remainder of the paper is organized as follows. Section 2 describes So and Shin's (1999) RMA and three alternative methods to construct confidence intervals for the persistent parameter estimate. In Section 3, we report our main empirical results and Section 4 concludes.

2 The Econometric Model

2.1 Recursive Mean Adjustment

Let p_t be the domestic price level, p_t^* be the foreign price level, and e_t be the nominal exchange rate as the unit price of the foreign currency in terms of the home currency. All variables are expressed in natural logarithms and are integrated processes of order 1. When PPP holds, there exists a cointegrating vector $[1 \quad 1 \quad -1]'$ for the vector $[p_t^* \quad e_t \quad p_t]'$, the log real exchange rate,

$s_t = p_t^* + e_t - p_t$, can be represented by a stationary AR process such as,

$$s_t = c + u_t, \tag{1}$$
$$u_t = \sum_{j=1}^{p} \rho_j u_{t-j} + \varepsilon_t,$$

where $\rho = \sum_{j=1}^{p} \rho_j$ is less than one in absolute value ($|\rho| < 1$) and ε_t is a mean-zero white noise process. Equivalently, the AR model (1) can be alternatively represented by,

$$s_t = c(1-\rho) + \sum_{j=1}^{p} \rho_j s_{t-j} + \varepsilon_t, \tag{2}$$

which implies the following augmented Dickey-Fuller form,

$$s_t = (1-\rho)c + \rho s_{t-1} + \sum_{j=1}^{k} \beta_j \Delta s_{t-j} + \varepsilon_t, \tag{3}$$

where $k = p - 1$, $\beta_j = -\sum_{s=j+1}^{p} \rho_s$, and $\rho = \sum_{j=1}^{p} \rho_j$ as previously defined.

Assuming that PPP holds, the persistence parameter ρ can be estimated by the conventional LS estimator. When $p = 1$, (1) can be written as,,

$$s_t = (1-\rho)c + \rho s_{t-1} + \varepsilon_t \tag{4}$$

By the Frisch-Waugh-Lovell theorem, (4) can be equivalently estimated by,

$$s_t - \bar{s} = \rho(s_{t-1} - \bar{s}) + \eta_t, \tag{5}$$

where $\bar{s} = T^{-1}\sum_{i=1}^{T} s_i$ is a sample mean and $\eta_t = \varepsilon_t - (1-\rho)c - (1-\rho)\bar{s}$. Note that ε_t thus η_t is correlated with the demeaned regressor $(s_{t-1} - \bar{s})$ because ε_t is correlated with s_i for $i = t, t+1, \cdots, T$, which is embedded in the regressor $(s_{t-1} - \bar{s})$ through $\bar{s}$. Since the exogeneity assumption fails, the LS estimator, $\hat{\rho}_{LS}$, is biased. The bias has an analytical representation and one can obtain the exactly mean-unbiased estimate by using a formula by Kendall (1954).

This paper corrects for the bias by employing an alternative method, the recursive mean adjustment (RMA), proposed by So and Shin (1999). The RMA method is computationally simple yet powerful and flexible enough to deal with higher order AR models. For this, rewrite (4) as,

$$s_t - \bar{s}_{t-1} = \rho(s_{t-1} - \bar{s}_{t-1}) + \xi_t, \tag{6}$$

where $\bar{s}_{t-1} = (t-1)^{-1} \sum_{i=1}^{t-1} s_i$ is the recursive mean and $\xi_t = \varepsilon_t - (1-\rho)c - (1-\rho)\bar{s}_{t-1}$. Since ε_t is orthogonal to the adjusted regressor $(s_{t-1} - \bar{s}_{t-1})$, the RMA estimator $\hat{\rho}_{RMA}$ significantly reduces the bias.

When $p = k + 1 > 2$, we follow a single-equation version of Choi *et al.*'s (2008) method. That is, we first estimate (3) by the LS and construct the following.

$$s_t^+ = (1-\rho)c + \rho s_{t-1} + \varepsilon_t^+, \tag{7}$$

where $s_t^+ = s_t - \sum_{j=1}^{k} \hat{\rho}_{j,LS} \Delta s_{t-j}$ and $\varepsilon_t^+ = \varepsilon_t - \sum_{j=1}^{k} (\hat{\rho}_{j,LS} - \rho_j) \Delta s_{t-j}$. Then, we apply RMA to (7),

$$s_t^+ - \bar{s}_{t-1} = \rho(s_{t-1} - \bar{s}_{t-1}) + \nu_t, \tag{8}$$

where $\nu_t = \varepsilon_t^+ + (1-\rho)c - (1-\rho)\bar{s}_{t-1}$. Finally, the RMA estimator $\hat{\rho}_{RMA}$ is obtained by,

$$\hat{\rho}_{RMA} = \frac{\sum_{i=2}^{T} (s_{t-1} - \bar{s}_{t-1})(s_t^+ - \bar{s}_{t-1})}{\sum_{i=2}^{T} (s_{t-1} - \bar{s}_{t-1})^2} \tag{9}$$

2.2 Constructing Confidence Intervals

Given the point estimate $\hat{\rho}_{RMA}$, it is important to obtain a reliable confidence interval for the estimate. We consider the following three popular approaches to compute confidence intervals: the asymptotic confidence interval, the percentile bootstrap confidence interval, and the bootstrap-t confidence interval.

It is not advisable to use the asymptotic confidence interval for $\hat{\rho}_{LS}$ because of its downward bias. So and Shin (1999), however, show that the asymptotic confidence interval for the RMA estimator has a very good coverage property via Monte Carlo simulations. The 90% asymptotic confidence interval for $\hat{\rho}_{RMA}$ is,

$$\left[\hat{\rho}_{RMA} - 1.645 \cdot se(\hat{\rho}_{RMA}),\ \hat{\rho}_{RMA} - 1.645 \cdot se(\hat{\rho}_{RMA})\right], \tag{10}$$

where $se(\hat{\rho}_{RMA}) = \hat{\sigma}(\hat{\rho}_{RMA}) / \left[\sum_{i=2}^{T}(s_{t-1} - \bar{s}_{t-1})^2\right]^{1/2}$ and $\hat{\sigma}(\hat{\rho}_{RMA})$ is the estimated standard error.

For the (nonparametric) percentile bootstrap confidence interval, let $\hat{F}$ be the empirical cumulative distribution function of $\hat{\rho}_{RMA}$ obtained from nonparametric bootstrap simulations. The 90% confidence interval is,

$$\left[\hat{F}_{0.05}^{-1},\ \hat{F}_{0.95}^{-1}\right] = \left[\hat{\rho}_{RMA,0.05},\ \hat{\rho}_{RMA,0.95}\right], \tag{11}$$

where $\hat{F}_{\alpha}^{-1}$ is the α percentile of the bootstrap distribution.

Finally, the bootstrap-t confidence interval (Efron and Tibshirani, 1993) is obtained as follows. Denote $\hat{Z}$ as the empirical cumulative distribution function of,

$$\hat{z}^i = \frac{\hat{\rho}_{RMA}^i - \hat{\rho}_{RMA}}{se(\hat{\rho}_{RMA}^i)}, \tag{12}$$

where $\hat{\rho}_{RMA}^i$ and $se(\hat{\rho}_{RMA}^i)$ are the RMA point estimate and the standard error from the i^{th} bootstrap sample. The 90% confidence interval is then obtained by,

$$\left[\hat{\rho}_{RMA} - \hat{z}_{0.95} \cdot se(\hat{\rho}_{RMA}),\ \hat{\rho}_{RMA} - \hat{z}_{0.05} \cdot se(\hat{\rho}_{RMA})\right], \tag{13}$$

where $\hat{z}_{\alpha}$ is the α percentile of the bootstrap distribution $\hat{Z}$.

3 Empirical Results

We consider CPI based real exchange rates for 21 industrialized countries. Our data set consists of quarterly observations from 1974:Q1 to 1998:Q4 for eurozone countries and from 1974:Q1 to 2005:Q4 for non-eurozone countries. The USA is taken as the numeraire (home) country and nominal exchange rates and CPIs are from the *International Financial Statistics*.

We start with the conventional ADF test for the real exchange rates. We select the number of lags (k) by the General-to-Specific rule as recommended by Ng and Perron (2001) for the ADF test. The test results are presented in Table 1. As can be seen from the table, the ADF test with the LS estimator (without bias correction) rejects the null of unit root for only 5 out of 20 countries at the 10% significance level. However, when we apply recursive mean adjustment (RMA) method to correct for the bias in the LS estimator the test results change dramatically. The null of unit root is now rejected for 16 out of 20 countries at the 10% significance level.[2]

So and Shin (1999) and Shin and So (2001) show that the RMA estimator is powerful and presents excellent coverage property yet computationally simple to obtain. Table 2 reports our estimates for the persistent parameter. To examine the performance of the RMA estimator ($\hat{\rho}_{RMA}$), we also report the LS estimator ($\hat{\rho}_{LS}$). We first note from the Table 2 that the RMA estimator yields significant bias-corrections. For all real exchange rates, persistent parameter estimates become greater with RMA. For example, the persistent parameter estimate increases from 0.920 (LS) to 0.940 (RMA) for the UK. The RMA estimator delivers effective bias correction that ranges from 0.009 (Canada) to 0.02 (UK) which is far from negligible.

Given the point estimate $\hat{\rho}_{RMA}$, it is important to obtain a reliable confidence interval for the estimate. Particularly for the case of highly persistent parameter estimates, confidence intervals provide useful information in exploring dynamics of the time-series of interest. It is well-known that the asymptotic confidence interval for $\hat{\rho}_{LS}$ performs very poorly (see Hansen 1999, for example). So and Shin (1999), however, show that the asymptotic confidence interval for the RMA estimator exhibits a very good coverage property via Monte Carlo simulations. To gauge the effectiveness of bias correction attained by the RMA estimator, we consider three alternative ways to compute confidence intervals. They are asymptotic confidence interval (CI_A), the percentile bootstrap confidence interval (CI_ρ), and the bootstrap-t confidence interval (CI_t).

The 90% confidence intervals we got from the percentile bootstrapping are narrow but upper bounds for the persistent parameter estimates are less than unity for all 21 countries. This does not conform to the results of unit-root tests with RMA since the upper bounds are too low with the percentile methods even for the countries where our unit-root test fails to reject the null. By contrast the bootstrap-t method returns higher lower bounds for the parameter estimates but now the upper bounds hit unity for almost all the countries. Only 2 out of 21 countries show less than unity upper bounds at the 90% confidence intervals with bootstrap-t method and the results are not consistent with the unit-root test results in Table 1.

However, we obtain the compact 90% *asymptotic* confidence intervals for $\hat{\rho}_{RMA}$ for 16 out of 20

[2]Higher power is obtained with RMA because the reduced-bias estimation right-shifts the critical values while the limiting distribution of ρ is not affected by RMA. See Shin and So (2001) for detailed explanations.

countries. These confidence intervals are also consistent with the results of the unit-root tests with RMA appeared in Table 1. It seems that the asymptotic confidence interval performs reasonably well in terms of both parsimony and efficiency even though it is computationally simple.

Murray and Papell (2002) claim that univariate approaches provide virtually no useful information on the size of real exchange rate half-lives since the confidence intervals for the point estimates are too wide and often the upper bounds are infinite. However, when we apply RMA to correct for the bias in the LS estimates in univariate ADF regressions, we obtain much tighter confidence intervals for the persistent parameter estimates with less than unity upper bounds. That is, by stark contrast to Murray and Papell (2002), our findings suggest that the univariate methods can provide useful information regarding the size of real exchange rate half-lives with more powerful but straightforward bias correction method of RMA.

4 Concluding Remarks

This paper revisits the empirical evidence on real exchange rate dynamics with recently developed RMA method. Using the current float quarterly real exchange rate data we find that the unit-root test with the RMA estimator rejects the null of unit root for 16 out of 20 industrialized countries while the conventional ADF tests rejects the null only for 5 countries. By stark contrast to Murray and Papell's (2002) claim, we find that the univariate RMA estimator and simple asymptotic confidence intervals can provide useful information regarding the real exchange rate dynamics.

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Table 1. Unit Root Tests: Short-Horizon Quarterly Real Exchange Rates

Country	Lag	ADF^{LS}	ADF^{RMA}
Australia	3	-2.525	-1.672*
Austria	3	-2.273	-1.856*
Belgium	3	-2.312	-1.827*
Canada	3	-2.023	-1.333
Denmark	3	-2.582*	-2.191†
Finland	3	-2.740*	-2.502†
France	1	-2.329	-1.749*
Germany	4	-2.596*	-2.285†
Greece	4	-2.233	-1.753*
Ireland	1	-2.467	-2.016†
Italy	3	-2.467	-2.202†
Japan	3	-2.263	-1.496
Netherlands	3	-2.367	-1.955†
New Zealand	3	-3.154†	-2.829‡
Norway	1	-2.297	-1.819*
Portugal	3	-1.682	-1.137
Spain	1	-1.955	-1.304
Sweden	3	-2.292	-1.813*
Switzerland	3	-2.832*	-2.353†
UK	1	-2.382	-1.788*

Note: i) Sample periods are 1973Q1-1998Q4 for the Euro-zone countries and 1973Q1-2004Q4 for the non Euro-zone countries. ii) ADF^{LS} and ADF^{RMA}refer to the augmented Dickey-Fuller unit root t-test with LS and RMA estimator, respectively, when an intercept is included. iii) The number of lags was chosen by the general-to-specific rule (Hall, 1994). iv) The asymptotic critical values for the ADF^{RMA} test were obtained from Shin and Soh (2001). v) $*$, †, and ‡ refer the cases that the null of unit root is rejected at the 10%, 5%, and 1% significance level.

Table 2. Recursive Mean Adjustment Estimates

Country	Lag	$\hat{\rho}_{LS}$	$\hat{\rho}_{RMA}$	CI$_A$	CI$_\rho$	CI$_t$
Australia	3	0.945	0.964	[0.929,0.999]*	[0.877,0.979]*	[0.949,1.000]
Austria	3	0.933	0.947	[0.901,0.994]*	[0.852,0.975]*	[0.916,1.000]
Belgium	3	0.938	0.953	[0.911,0.995]*	[0.866,0.976]*	[0.926,1.000]
Canada	3	0.973	0.982	[0.960,1.000]	[0.925,0.992]*	[0.971,1.000]
Denmark	3	0.930	0.943	[0.901,0.986]*	[0.863,0.969]*	[0.913,1.000]
Finland	3	0.908	0.921	[0.869,0.973]*	[0.826,0.959]*	[0.878,0.991]*
France	1	0.930	0.947	[0.898,0.997]*	[0.847,0.972]*	[0.918,1.000]
Germany	4	0.900	0.918	[0.860,0.977]*	[0.798,0.958]*	[0.873,1.000]
Greece	4	0.932	0.949	[0.902,0.997]*	[0.843,0.977]*	[0.919,1.000]
Ireland	1	0.905	0.923	[0.860,0.986]*	[0.804,0.959]*	[0.881,1.000]
Italy	3	0.918	0.931	[0.880,0.983]*	[0.833,0.966]*	[0.892,1.000]
Japan	3	0.950	0.967	[0.932,1.000]	[0.881,0.983]*	[0.951,1.000]
Netherlands	3	0.924	0.940	[0.890,0.991]*	[0.839,0.970]*	[0.906,1.000]
New Zealand	3	0.913	0.927	[0.885,0.969]*	[0.852,0.955]*	[0.896,0.985]*
Norway	1	0.933	0.947	[0.899,0.995]*	[0.854,0.972]*	[0.917,1.000]
Portugal	3	0.958	0.972	[0.932,1.000]	[0.871,0.990]*	[0.952,1.000]
Spain	1	0.951	0.967	[0.926,1.000]	[0.872,0.986]*	[0.947,1.000]
Sweden	3	0.953	0.964	[0.931,0.997]*	[0.895,0.982]*	[0.943,1.000]
Switzerland	3	0.915	0.932	[0.885,0.980]*	[0.842,0.960]*	[0.901,1.000]
UK	1	0.920	0.940	[0.885,0.995]*	[0.838,0.964]*	[0.911,1.000]

Note: i) Sample periods are 1973Q1-1998Q4 for the Euro-zone countries and 1973Q1-2004Q4 for the non Euro-zone countries. ii) k is chosen by the general-to-specific rule (Hall, 1994). iii) $\hat{\rho}_{LS}$ and $\hat{\rho}_{RMA}$ refer to the least squares ρ estimate and the recursive mean adjustment ρ estimate, respectively. iv) CI$_A$ denotes the 90% asymptotic confidence interval for $\hat{\rho}_{RMA}$ using the normal approximation. CI$_\rho$ and CI$_t$ refer to the 90% nonparametric percentile bootstrap confidence interval and the 90% nonparametric bootstrap-t confidence interval obtained from

10,000 bootstrap replications from the empirical distribution at the RMA point estimates (Efron and Tibshirani, 1993). Superscript $*$ refers to the cases where the upper baound is less than unity at the 90% confidence interval.